Λ_c polarimetry using the dominant hadronic mode — supplemental material

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TABLE OF CONTENTS

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Λ_c^+ polarimetry using the dominant hadronic mode The polarimeter vector field for multibody decays of a spin-half baryon is introduced as a generalisation of the baryon asymmetry parameters. Using a recent amplitude analysis of the $\Lambda_c^+ \to pK^-\pi^+$ decay performed at the LHCb experiment, we compute the distribution of the kinematic-dependent polarimeter vector for this process in the space of Mandelstam variables to express the polarised decay rate in a model-agnostic form. The obtained representation can facilitate polarisation measurements of the Λ_c^+ baryon and eases inclusion of the $\Lambda_c^+ \to pK^-\pi^+$ decay mode in hadronic amplitude analyses.

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This website shows all analysis results that led to the publication of LHCb-PAPER-2022-044. More information on this publication can be found on the following pages:

- Publication on JHEP: J. High Energ. Phys. 2023, 228 (2023)
- Publication on arXiv: arXiv:2301.07010
- Record on CDS: cds.cern.ch/record/2838694
- Record for the source code on Zenodo: 10.5281/zenodo.7544989
- Archived documentation on GitLab Pages: lc2pkpi-polarimetry.docs.cern.ch
- Archived repository on CERN GitLab: gitlab.cern.ch/polarimetry/Lc2pKpi
- Active repository on GitHub containing discussions: github.com/ComPWA/polarimetry

Behind SSO login (LHCb members only)

- LHCb TWiki page: twiki.cern.ch/twiki/bin/viewauth/LHCbPhysics/PolarimetryLc2pKpi
- Charm WG meeting: indico.cern.ch/event/1187317
- RC approval presentation: indico.cern.ch/event/1213570
- Silent approval to submit: indico.cern.ch/event/1242323

Note

This document is a PDF rendering of the supplemental material hosted behind SSO-login on lc2pkpi-polarimetry.docs.cern.ch. Go to this webpage for a more extensive and interactive experience.

CHAPTER

ONE

PYTHON PACKAGE

pypi package 0.0.11 python 3.8 | 3.9 | 3.10 | 3.11 | 3.12

Each of the pages contain code examples for how to reproduce the results with the Python package hosted at github.com/ComPWA/polarimetry. However, to quickly get import the model in another package, it is possible to install the package from PyPI:

```
pip install polarimetry-lc2pkpi
```

Each of the models can then simply be imported as

$$\sum_{\lambda_0=-1/2}^{1/2}\sum_{\lambda_1=-1/2}^{1/2}\left|\sum_{\lambda_0'=-1/2}^{1/2}\sum_{\lambda_1'=-1/2}^{1/2} A^{1}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{1(1)}\right) + A^{2}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{2(1)}\right) + A^{3}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda}\right.$$

For more examples of how to use the codebase, see the following pages.

1.1 Default amplitude model

1.1.1 Resonances and LS-scheme

Particle definitions for Λ_c^+ and p, π^+, K^- in the sequential order.

index	name	LaTeX	J^P	mass (MeV)	width (MeV)
0	`Lambda_c+`	Λ_c^+	$\frac{1}{2}^+$	2,286	0
1	`p`	p	$\frac{1}{2}^+$	938	0
2	`pi+`	π^+	0^-	139	0
3	`K-`	K^-	0^-	493	0
`Sigma-`	Σ^-	$\frac{1}{2}^+$	1,189	0	

Particle definitions as defined in `particle-definitions.yaml`:

name	LaTeX	J^P	mass (MeV)	width (MeV)
`Lambda_c+`	Λ_c^+	$\frac{1}{2}^+$	2,286	0
`p`	p	$\frac{1}{2}^+$	938	0
`pi+`	π^+	0^-	139	0
`K-`	K^-	0^-	493	0
`L(1405)`	$\Lambda(1405)$	$\frac{1}{2}^-$	1,405	50
`L(1520)`	$\Lambda(1520)$	$\frac{3}{2}^-$	1,519	15
`L(1600)`	$\Lambda(1600)$	$\frac{1}{2}^+$	1,630	250
`L(1670)`	$\Lambda(1670)$	$\frac{1}{2}^-$	1,670	30
`L(1690)`	$\Lambda(1690)$	$\frac{3}{2}^-$	1,690	70
`L(1800)`	$\Lambda(1800)$	$\frac{1}{2}^-$	1,800	300
`L(1810)`	$\Lambda(1810)$	$\frac{1}{2}^+$	1,810	150
`L(2000)`	$\Lambda(2000)$	$\frac{1}{2}^-$	2,000	210
`D(1232)`	$\Delta(1232)$	$\frac{3}{2}^+$	1,232	117
`D(1600)`	$\Delta(1600)$	$\frac{3}{2}^+$	1,640	300
`D(1620)`	$\Delta(1620)$	$\frac{1}{2}^-$	1,620	130
`D(1700)`	$\Delta(1700)$	$\frac{3}{2}^-$	1,690	380
`K(700)`	$K(700)$	0^+	824	478
`K(892)`	$K(892)$	1^-	895	47
`K(1410)`	$K(1410)$	1^-	1,421	236
`K(1430)`	$K(1430)$	0^+	1,375	190

See also

Amplitude model with LS-couplings (page 37)

Most models work take the **minimal L-value** in each LS-coupling (only model 17 works in the full LS-basis. The generated LS-couplings look as follows:

Only minimum LS (12)	All LS-couplings (26)
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+\right) K^-$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+\right) K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+\right) K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+\right) K^-$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(700) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(700) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(700) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right) p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right) p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right) p$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right) p$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right) p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-\right) p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\right) \pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\right) \pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\right) \pi^+$

Or with J^P-values:

Only minimum LS (12)	All LS-couplings (26)
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1232) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1232) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (\Delta(1232) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1600) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1600) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (\Delta(1600) [\frac{3}{2}^+] \xrightarrow[S=1/2]{L=1} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1700) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} p [\frac{1}{2}^+] \pi^+ [0$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Delta(1700) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (\Delta(1700) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} p [\frac{1}{2}^+] \pi^+ [0^-]) K^- [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(700) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-]$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(700) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (K(700) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(892) [1^-] \xrightarrow[S=0]{L=1} \pi^+ [0^-] K^- [0^-]$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(892) [1^-] \xrightarrow[S=0]{L=1} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (K(892) [1^-] \xrightarrow[S=0]{L=1} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (K(892) [1^-] \xrightarrow[S=0]{L=1} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (K(892) [1^-] \xrightarrow[S=0]{L=1} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(1430) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (K(1430) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (K(1430) [0^+] \xrightarrow[S=0]{L=0} \pi^+ [0^-] K^- [0^-]) p [\frac{1}{2}^+]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1405) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1405) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (\Lambda(1405) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Lambda(1520) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Lambda(1520) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (\Lambda(1520) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1600) [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1600) [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (\Lambda(1600) [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1670) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(1670) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (\Lambda(1670) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Lambda(1690) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=1} (\Lambda(1690) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=3/2]{L=2} (\Lambda(1690) [\frac{3}{2}^-] \xrightarrow[S=1/2]{L=2} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(2000) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}$	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=0} (\Lambda(2000) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$
	$\Lambda_c^+ [\frac{1}{2}^+] \xrightarrow[S=1/2]{L=1} (\Lambda(2000) [\frac{1}{2}^-] \xrightarrow[S=1/2]{L=0} K^- [0^-] p [\frac{1}{2}^+]) \pi^+ [0^-]$

1.1.2 Amplitude

Spin-alignment amplitude

The full intensity of the amplitude model is obtained by summing the following aligned amplitude over all helicity values λ_i in the initial state 0 and final states $1, 2, 3$:

$$\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^{1}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{1(1)}\right) + A^{2}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{2(1)}\right) + A^{3}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{3(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}$$

Note that we simplified notation here: the amplitude indices for the spinless states are not rendered and their corresponding Wigner-d alignment functions are simply 1.

The relevant $\zeta^i_{j(k)}$ angles are *defined as* (page 29):

$$\begin{aligned}
\zeta^{0}_{1(1)} &= 0 \\
\zeta^{1}_{1(1)} &= 0 \\
\zeta^{0}_{2(1)} &= -\operatorname{acos}\left(\frac{-2m_0^2(-m_1^2-m_2^2+\sigma_3)+(m_0^2+m_1^2-\sigma_1)(m_0^2+m_2^2-\sigma_2)}{\sqrt{\lambda(m_0^2,m_2^2,\sigma_2)}\sqrt{\lambda(m_0^2,\sigma_1,m_1^2)}}\right) \\
\zeta^{1}_{2(1)} &= \operatorname{acos}\left(\frac{2m_1^2(-m_0^2-m_3^2+\sigma_3)+(m_0^2+m_1^2-\sigma_1)(-m_1^2-m_3^2+\sigma_2)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_2,m_1^2,m_3^2)}}\right) \\
\zeta^{0}_{3(1)} &= \operatorname{acos}\left(\frac{-2m_0^2(-m_1^2-m_3^2+\sigma_2)+(m_0^2+m_1^2-\sigma_1)(m_0^2+m_3^2-\sigma_3)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(m_0^2,\sigma_3,m_3^2)}}\right) \\
\zeta^{1}_{3(1)} &= -\operatorname{acos}\left(\frac{2m_1^2(-m_0^2-m_2^2+\sigma_2)+(m_0^2+m_1^2-\sigma_1)(-m_1^2-m_2^2+\sigma_3)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right)
\end{aligned}$$

Sub-system amplitudes

$$\begin{aligned}
A^{1}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} -\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} \mathcal{H}^{\mathrm{production}}_{K(892),\lambda_R,-\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} -\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{Bugg}}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(1430),} \\
A^{2}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} -\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,1}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,-\frac{1}{2}} \mathcal{H}^{\mathrm{production}}_{\Lambda(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} -\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1600),0} \\
A^{3}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),-\frac{1}{2},0} \mathcal{H}^{\mathrm{production}}_{\Delta(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1600),-\frac{1}{2}} \\
A^{1}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} \mathcal{H}^{\mathrm{production}}_{K(892),\lambda_R,\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} \delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{Bugg}}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(1430),0,0} \\
A^{2}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,1}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,\frac{1}{2}} \mathcal{H}^{\mathrm{production}}_{\Lambda(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1600),0,\frac{1}{2}} \\
A^{3}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),\frac{1}{2},0} \mathcal{H}^{\mathrm{production}}_{\Delta(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1600),\frac{1}{2},0} \\
A^{1}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} -\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} \mathcal{H}^{\mathrm{production}}_{K(892),\lambda_R,-\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} -\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{Bugg}}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(1430),0,0} \\
A^{2}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} -\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,1}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,-\frac{1}{2}} \mathcal{H}^{\mathrm{production}}_{\Lambda(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} -\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1600),0,-} \\
A^{3}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),-\frac{1}{2},0} \mathcal{H}^{\mathrm{production}}_{\Delta(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1600),-\frac{1}{2},0} \\
A^{1}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \delta_{\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} \mathcal{H}^{\mathrm{production}}_{K(892),\lambda_R,\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} \delta_{\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{Bugg}}\left(\sigma_1\right) \mathcal{H}^{\mathrm{decay}}_{K(1430),0,0} \\
A^{2}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,1}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,\frac{1}{2}} \mathcal{H}^{\mathrm{production}}_{\Lambda(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,0}\left(\sigma_2\right) \mathcal{H}^{\mathrm{decay}}_{\Lambda(1600),0,\frac{1}{2}} \\
A^{3}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),\frac{1}{2},0} \mathcal{H}^{\mathrm{production}}_{\Delta(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,1}\left(\sigma_3\right) \mathcal{H}^{\mathrm{decay}}_{\Delta(1600),\frac{1}{2},0}
\end{aligned}$$

The θ_{ij} angles are *defined as* (page 29):

$$
\begin{aligned}
\theta_{23} &= \operatorname{acos}\left(\frac{2\sigma_1(-m_1^2-m_2^2+\sigma_3)-(m_0^2-m_1^2-\sigma_1)(m_2^2-m_3^2+\sigma_1)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_1,m_2^2,m_3^2)}}\right) \\
\theta_{31} &= \operatorname{acos}\left(\frac{2\sigma_2(-m_2^2-m_3^2+\sigma_1)-(m_0^2-m_2^2-\sigma_2)(-m_1^2+m_3^2+\sigma_2)}{\sqrt{\lambda(m_0^2,m_2^2,\sigma_2)}\sqrt{\lambda(\sigma_2,m_3^2,m_1^2)}}\right) \\
\theta_{12} &= \operatorname{acos}\left(\frac{2\sigma_3(-m_1^2-m_3^2+\sigma_2)-(m_0^2-m_3^2-\sigma_3)(m_1^2-m_2^2+\sigma_3)}{\sqrt{\lambda(m_0^2,m_3^2,\sigma_3)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right)
\end{aligned}
$$

Definitions for the ϕ_{ij} angles can be found under *DPD angles* (page 29).

1.1.3 Parameter definitions

Parameter values are provided in `model-definitions.yaml`, but the **keys** of the helicity couplings have to remapped to the helicity **symbols** that are used in this amplitude model. The function `parameter_key_to_symbol()` (page 50) implements this remapping, following the supplementary material of [1]. It is asserted below that:

1. the keys are mapped to symbols that exist in the default amplitude model
2. all parameter symbols in the default amplitude model have a value assigned to them.

Helicity coupling values

Production couplings

$$
\begin{aligned}
\mathcal{H}^\text{production}_{K(892), -1, -\frac{1}{2}} &= 1.192614 - 1.025814i \\
\mathcal{H}^\text{production}_{\Lambda(1405), -\frac{1}{2}, 0} &= -4.572486 + 3.190144i \\
\mathcal{H}^\text{production}_{\Lambda(1520), -\frac{1}{2}, 0} &= 0.293998 + 0.044324i \\
\mathcal{H}^\text{production}_{\Lambda(1600), -\frac{1}{2}, 0} &= -4.840649 - 3.082786i \\
\mathcal{H}^\text{production}_{\Lambda(1670), -\frac{1}{2}, 0} &= -0.339585 - 0.144678i \\
\mathcal{H}^\text{production}_{\Lambda(1690), -\frac{1}{2}, 0} &= -0.385772 - 0.110235i \\
\mathcal{H}^\text{production}_{\Lambda(2000), -\frac{1}{2}, 0} &= -8.014857 - 7.614006i \\
\mathcal{H}^\text{production}_{\Delta(1232), -\frac{1}{2}, 0} &= -6.778191 + 3.051805i \\
\mathcal{H}^\text{production}_{\Delta(1600), -\frac{1}{2}, 0} &= 11.401585 - 3.125511i \\
\mathcal{H}^\text{production}_{\Delta(1700), -\frac{1}{2}, 0} &= -10.37828 - 1.434872i \\
\mathcal{H}^\text{production}_{K(700), 0, \frac{1}{2}} &= 0.068908 + 2.521444i \\
\mathcal{H}^\text{production}_{K(892), 0, \frac{1}{2}} &= -0.727145 - 4.155027i \\
\mathcal{H}^\text{production}_{K(1430), 0, \frac{1}{2}} &= -6.71516 + 10.479411i \\
\mathcal{H}^\text{production}_{K(700), 0, -\frac{1}{2}} &= -2.68563 + 0.03849i \\
\mathcal{H}^\text{production}_{K(892), 0, -\frac{1}{2}} &= 1 + 0i \\
\mathcal{H}^\text{production}_{K(1430), 0, -\frac{1}{2}} &= 0.219754 + 8.741196i \\
\mathcal{H}^\text{production}_{\Lambda(1405), \frac{1}{2}, 0} &= 10.44608 + 2.787441i \\
\mathcal{H}^\text{production}_{\Lambda(1520), \frac{1}{2}, 0} &= -0.160687 + 1.498833i \\
\mathcal{H}^\text{production}_{\Lambda(1600), \frac{1}{2}, 0} &= 6.971233 - 0.842435i \\
\mathcal{H}^\text{production}_{\Lambda(1670), \frac{1}{2}, 0} &= -0.570978 + 1.011833i \\
\mathcal{H}^\text{production}_{\Lambda(1690), \frac{1}{2}, 0} &= -2.730592 - 0.353613i \\
\mathcal{H}^\text{production}_{\Lambda(2000), \frac{1}{2}, 0} &= -4.336255 - 3.796192i \\
\mathcal{H}^\text{production}_{\Delta(1232), \frac{1}{2}, 0} &= -12.987193 + 4.528336i \\
\mathcal{H}^\text{production}_{\Delta(1600), \frac{1}{2}, 0} &= 6.729211 - 1.002383i \\
\mathcal{H}^\text{production}_{\Delta(1700), \frac{1}{2}, 0} &= -12.874102 - 2.10557i \\
\mathcal{H}^\text{production}_{K(892), 1, \frac{1}{2}} &= -3.141446 - 3.29341i
\end{aligned}
$$

Decay couplings

$$
\begin{aligned}
\mathcal{H}^\text{decay}_{K(892),0,0} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1405),0,-\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1520),0,-\frac{1}{2}} &= -1\\
\mathcal{H}^\text{decay}_{\Lambda(1600),0,-\frac{1}{2}} &= -1\\
\mathcal{H}^\text{decay}_{\Lambda(1670),0,-\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1690),0,-\frac{1}{2}} &= -1\\
\mathcal{H}^\text{decay}_{\Lambda(2000),0,-\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1232),-\frac{1}{2},0} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1600),-\frac{1}{2},0} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1700),-\frac{1}{2},0} &= -1\\
\mathcal{H}^\text{decay}_{K(700),0,0} &= 1\\
\mathcal{H}^\text{decay}_{K(1430),0,0} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1405),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1520),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1600),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1670),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(1690),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Lambda(2000),0,\frac{1}{2}} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1232),\frac{1}{2},0} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1600),\frac{1}{2},0} &= 1\\
\mathcal{H}^\text{decay}_{\Delta(1700),\frac{1}{2},0} &= 1
\end{aligned}
$$

Non-coupling parameters

$$
\begin{aligned}
R_{\mathrm{res}} &= 1.5 \\
R_{\Lambda_c} &= 5 \\
\Gamma_{D(1232)} &= 0.117 \\
\Gamma_{D(1600)} &= 0.3 \\
\Gamma_{D(1700)} &= 0.38 \\
\Gamma_{K(1430)} &= 0.19 \\
\Gamma_{K(700)} &= 0.47800000000000004 \\
\Gamma_{K(892)} &= 0.047299999999999995 \\
\Gamma_{L(1405)\to K^- p} &= 0.0505 \\
\Gamma_{L(1405)\to \pi^+ \Sigma^-} &= 0.0505 \\
\Gamma_{L(1520)} &= 0.015195 \\
\Gamma_{L(1600)} &= 0.25 \\
\Gamma_{L(1670)} &= 0.03 \\
\Gamma_{L(1690)} &= 0.07 \\
\Gamma_{L(2000)} &= 0.17926 \\
\gamma_{K(1430)} &= 0.020981 \\
\gamma_{K(700)} &= 0.94106 \\
m_{0} &= 2.28646 \\
m_{1} &= 0.938272046 \\
m_{2} &= 0.13957018 \\
m_{3} &= 0.49367700000000003 \\
m_{D(1232)} &= 1.232 \\
m_{D(1600)} &= 1.6400000000000001 \\
m_{D(1700)} &= 1.69 \\
m_{K(1430)} &= 1.375 \\
m_{K(700)} &= 0.8240000000000001 \\
m_{K(892)} &= 0.8955000000000001 \\
m_{K-} &= 0.49367700000000003 \\
m_{L(1405)} &= 1.4051 \\
m_{L(1520)} &= 1.518467 \\
m_{L(1600)} &= 1.6300000000000001 \\
m_{L(1670)} &= 1.67 \\
m_{L(1690)} &= 1.69 \\
m_{L(2000)} &= 1.98819 \\
m_{Lambda_c+} &= 2.28646 \\
m_{\Sigma^-} &= 1.1893699999999998 \\
m_{\pi^+} &= 0.13957018 \\
m_{pi+} &= 0.13957018 \\
m_{p} &= 0.938272046
\end{aligned}
$$

1.2 Cross-check with LHCb data

1.2.1 Lineshape comparison

We compute a few lineshapes for the following point in phase space and compare it with the values from [1]:

```
{'costhetap': -0.9949949110827053,
 'm2kpi': 0.7980703453578917,
 'm2pk': 3.6486261122281745,
 'phikpi': -0.4,
 'phip': -0.3}
```

The lineshapes are computed for the following decay chains:

$$\begin{aligned}
\Lambda_c^+ &\xrightarrow[S=1/2]{L=0} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+ K^-\right) p \\
\Lambda_c^+ &\xrightarrow[S=1/2]{L=0} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^- p\right) \pi^+ \\
\Lambda_c^+ &\xrightarrow[S=3/2]{L=1} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^- p\right) \pi^+
\end{aligned}$$

```
{'BW_K(892)_p^1_q^0': '(2.1687201455088894+23.58225917009096j)',
 'BW_L(1405)_p^0_q^0': '(-0.5636481410171861+0.13763637759224928j)',
 'BW_L(1690)_p^2_q^1': '(-1.5078327158518026+0.9775036395061584j)'}
```

$$\begin{gathered}
2.16872014550901 + 23.5822591700909i \\
-0.563648141017186 + 0.137636377592249i \\
-1.5078327158518 + 0.977503639506157i
\end{gathered}$$

Tip

These values are **equal up to 13 decimals**.

1.2.2 Amplitude comparison

The amplitude for each decay chain and each outer state helicity combination are evaluated on the following point in phase space:

$$\begin{aligned}
\theta_{23} &= 1.821341166520149 \\
\theta_{31} &= 1.8038351483715633 \\
\theta_{12} &= 1.1139045236042229 \\
\zeta^0_{1(1)} &= 0.0 \\
\zeta^1_{1(1)} &= 0.0 \\
\zeta^0_{2(1)} &= -2.0777687076712614 \\
\zeta^1_{2(1)} &= 0.22583331080386268 \\
\zeta^0_{3(1)} &= 2.6540796539955838 \\
\zeta^1_{3(1)} &= -0.5594175047790548 \\
\sigma_1 &= 0.7980703453578917 \\
\sigma_2 &= 3.6486261122281745 \\
\sigma_3 &= 1.9247541217931925
\end{aligned}$$

Default model

Tip

Computed amplitudes are equal to LHCb amplitudes up to **13 decimals**.

	Computed	Expected	Difference
ArD(1232)1	$\mathcal{H}^{\text{production}}_{\Delta(1232),-\frac{1}{2},0}$		
A++	-0.488498+0.517710j	-0.488498+0.517710j	3.11e-14
A+-	0.894898-0.948412j	0.894898-0.948412j	7.61e-15
A-+	0.121490-0.128755j	0.121490-0.128755j	1.80e-14
A--	-0.222563+0.235872j	-0.222563+0.235872j	6.14e-15
ArD(1232)2	$\mathcal{H}^{\text{production}}_{\Delta(1232),\frac{1}{2},0}$		
A++	-0.222563+0.235872j	-0.222563+0.235872j	6.14e-15
A+-	-0.121490+0.128755j	-0.121490+0.128755j	1.80e-14
A-+	-0.894898+0.948412j	-0.894898+0.948412j	7.61e-15
A--	-0.488498+0.517710j	-0.488498+0.517710j	3.11e-14
ArD(1600)1	$\mathcal{H}^{\text{production}}_{\Delta(1600),-\frac{1}{2},0}$		
A++	0.289160+0.081910j	0.289160+0.081910j	3.11e-14
A+-	-0.529724-0.150054j	-0.529724-0.150054j	7.48e-15
A-+	-0.071915-0.020371j	-0.071915-0.020371j	1.82e-14
A--	0.131743+0.037319j	0.131743+0.037319j	5.71e-15
ArD(1600)2	$\mathcal{H}^{\text{production}}_{\Delta(1600),\frac{1}{2},0}$		
A++	0.131743+0.037319j	0.131743+0.037319j	5.71e-15
A+-	0.071915+0.020371j	0.071915+0.020371j	1.82e-14
A-+	0.529724+0.150054j	0.529724+0.150054j	7.48e-15
A--	0.289160+0.081910j	0.289160+0.081910j	3.11e-14
ArD(1700)1	$\mathcal{H}^{\text{production}}_{\Delta(1700),-\frac{1}{2},0}$		
A++	-0.018885-0.001757j	-0.018885-0.001757j	3.18e-13
A+-	0.315695+0.029366j	0.315695+0.029366j	2.04e-14
A-+	0.004697+0.000437j	0.004697+0.000437j	3.30e-13
A--	-0.078514-0.007303j	-0.078514-0.007303j	7.22e-15
ArD(1700)2	$\mathcal{H}^{\text{production}}_{\Delta(1700),\frac{1}{2},0}$		
A++	0.078514+0.007303j	0.078514+0.007303j	7.22e-15
A+-	0.004697+0.000437j	0.004697+0.000437j	3.30e-13
A-+	0.315695+0.029366j	0.315695+0.029366j	2.04e-14
A--	0.018885+0.001757j	0.018885+0.001757j	3.18e-13
ArK(892)1	$\mathcal{H}^{\text{production}}_{K(892),0,-\frac{1}{2}}$		
A++	-0.537695-5.846793j	-0.537695-5.846793j	5.00e-15
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.000000+0.000000j	0.000000+0.000000j	
ArK(892)2	$\mathcal{H}^{\text{production}}_{K(892),-1,-\frac{1}{2}}$		
A++	-0.000000+0.000000j	0.000000+0.000000j	
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	1.485636+16.154534j	1.485636+16.154534j	3.20e-15
A--	0.000000+0.000000j	0.000000+0.000000j	
ArK(892)3	$\mathcal{H}^{\text{production}}_{K(892),1,\frac{1}{2}}$		
A++	-0.000000+0.000000j	0.000000+0.000000j	
A+-	-1.485636-16.154534j	-1.485636-16.154534j	3.20e-15
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.000000+0.000000j	0.000000+0.000000j	
ArK(892)4	$\mathcal{H}^{\text{production}}_{K(892),0,\frac{1}{2}}$		
A++	-0.000000+0.000000j	0.000000+0.000000j	
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-0.537695-5.846793j	-0.537695-5.846793j	5.00e-15
ArK(1430)1	$\mathcal{H}^{\text{production}}_{K(1430),0,\frac{1}{2}}$		

continues on next page

Table 1.1 – continued from previous page

	Computed	Expected	Difference
A++	-0.000000+0.000000j	0.000000+0.000000j	
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.909456+0.072819j	0.909456+0.072819j	1.22e-16
ArK(1430)2	$\mathcal{H}^\text{production}_{K(1430),0,-\frac{1}{2}}$		
A++	0.909456+0.072819j	0.909456+0.072819j	1.22e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.000000+0.000000j	0.000000+0.000000j	
ArK(700)1	$\mathcal{H}^\text{production}_{K(700),0,\frac{1}{2}}$		
A++	-0.000000+0.000000j	0.000000+0.000000j	
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-1.708879+3.380634j	-1.708879+3.380634j	4.97e-16
ArK(700)2	$\mathcal{H}^\text{production}_{K(700),0,-\frac{1}{2}}$		
A++	-1.708879+3.380634j	-1.708879+3.380634j	4.97e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.000000+0.000000j	0.000000+0.000000j	
ArL(1405)1	$\mathcal{H}^\text{production}_{\Lambda(1405),-\frac{1}{2},0}$		
A++	-0.412613+0.100755j	-0.412613+0.100755j	1.49e-15
A+-	-0.256372+0.062603j	-0.256372+0.062603j	3.06e-15
A-+	-0.242818+0.059293j	-0.242818+0.059293j	1.40e-15
A--	-0.150872+0.036841j	-0.150872+0.036841j	3.06e-15
ArL(1405)2	$\mathcal{H}^\text{production}_{\Lambda(1405),\frac{1}{2},0}$		
A++	-0.150872+0.036841j	-0.150872+0.036841j	3.06e-15
A+-	0.242818-0.059293j	0.242818-0.059293j	1.40e-15
A-+	0.256372-0.062603j	0.256372-0.062603j	3.06e-15
A--	-0.412613+0.100755j	-0.412613+0.100755j	1.49e-15
ArL(1520)1	$\mathcal{H}^\text{production}_{\Lambda(1520),-\frac{1}{2},0}$		
A++	0.257632-0.288056j	0.257632-0.288056j	1.52e-14
A+-	0.731594-0.817988j	0.731594-0.817988j	2.23e-14
A-+	0.151613-0.169517j	0.151613-0.169517j	1.51e-14
A--	0.430534-0.481376j	0.430534-0.481376j	2.22e-14
ArL(1520)2	$\mathcal{H}^\text{production}_{\Lambda(1520),\frac{1}{2},0}$		
A++	-0.430534+0.481376j	-0.430534+0.481376j	2.22e-14
A+-	0.151613-0.169517j	0.151613-0.169517j	1.51e-14
A-+	0.731594-0.817988j	0.731594-0.817988j	2.25e-14
A--	-0.257632+0.288056j	-0.257632+0.288056j	1.52e-14
ArL(1600)1	$\mathcal{H}^\text{production}_{\Lambda(1600),-\frac{1}{2},0}$		
A++	-0.385436+0.424707j	-0.385436+0.424707j	1.17e-15
A+-	0.382669-0.421658j	0.382669-0.421658j	3.88e-15
A-+	-0.226825+0.249935j	-0.226825+0.249935j	1.38e-15
A--	0.225196-0.248141j	0.225196-0.248141j	3.64e-15
ArL(1600)2	$\mathcal{H}^\text{production}_{\Lambda(1600),\frac{1}{2},0}$		
A++	-0.225196+0.248141j	-0.225196+0.248141j	3.68e-15
A+-	-0.226825+0.249935j	-0.226825+0.249935j	1.46e-15
A-+	0.382669-0.421658j	0.382669-0.421658j	3.88e-15
A--	0.385436-0.424707j	0.385436-0.424707j	1.17e-15
ArL(1670)1	$\mathcal{H}^\text{production}_{\Lambda(1670),-\frac{1}{2},0}$		
A++	-0.846639+0.064025j	-0.846639+0.064025j	1.18e-15

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Table 1.1 – continued from previous page

	Computed	Expected	Difference
A+-	-0.526049+0.039781j	-0.526049+0.039781j	2.96e-15
A-+	-0.498237+0.037678j	-0.498237+0.037678j	1.22e-15
A--	-0.309574+0.023411j	-0.309574+0.023411j	3.24e-15
ArL(1670)2	$\mathcal{H}^\mathrm{production}_{\Lambda(1670),\frac{1}{2},0}$		
A++	-0.309574+0.023411j	-0.309574+0.023411j	3.24e-15
A+-	0.498237-0.037678j	0.498237-0.037678j	1.22e-15
A-+	0.526049-0.039781j	0.526049-0.039781j	2.96e-15
A--	-0.846639+0.064025j	-0.846639+0.064025j	1.18e-15
ArL(1690)1	$\mathcal{H}^\mathrm{production}_{\Lambda(1690),-\frac{1}{2},0}$		
A++	0.232446-0.150691j	0.232446-0.150691j	1.63e-14
A+-	0.660073-0.427915j	0.660073-0.427915j	2.30e-14
A-+	0.136791-0.088680j	0.136791-0.088680j	1.62e-14
A--	0.388445-0.251823j	0.388445-0.251823j	2.29e-14
ArL(1690)2	$\mathcal{H}^\mathrm{production}_{\Lambda(1690),\frac{1}{2},0}$		
A++	-0.388445+0.251823j	-0.388445+0.251823j	2.31e-14
A+-	0.136791-0.088680j	0.136791-0.088680j	1.62e-14
A-+	0.660073-0.427915j	0.660073-0.427915j	2.32e-14
A--	-0.232446+0.150691j	-0.232446+0.150691j	1.63e-14
ArL(2000)1	$\mathcal{H}^\mathrm{production}_{\Lambda(2000),-\frac{1}{2},0}$		
A++	1.072514+1.195841j	1.072514+1.195841j	1.47e-15
A+-	0.666394+0.743022j	0.666394+0.743022j	2.69e-15
A-+	0.631162+0.703738j	0.631162+0.703738j	1.34e-15
A--	0.392165+0.437260j	0.392165+0.437260j	3.03e-15
ArL(2000)2	$\mathcal{H}^\mathrm{production}_{\Lambda(2000),\frac{1}{2},0}$		
A++	0.392165+0.437260j	0.392165+0.437260j	3.03e-15
A+-	-0.631162-0.703738j	-0.631162-0.703738j	1.34e-15
A-+	-0.666394-0.743022j	-0.666394-0.743022j	2.69e-15
A--	1.072514+1.195841j	1.072514+1.195841j	1.47e-15

LS-model

Tip

Computed amplitudes are equal to LHCb amplitudes up to **13 decimals**.

	Computed	Expected	Difference
ArD(1232)1	$\mathcal{H}^\mathrm{LS,production}_{\Delta(1232),1,\frac{3}{2}}$		
A++	0.502796-0.532862j	0.502796-0.532862j	1.94e-14
A+-	-0.546882+0.579585j	-0.546882+0.579585j	5.67e-15
A-+	0.546882-0.579585j	0.546882-0.579585j	5.67e-15
A--	0.502796-0.532862j	0.502796-0.532862j	1.93e-14
ArD(1232)2	$\mathcal{H}^\mathrm{LS,production}_{\Delta(1232),2,\frac{3}{2}}$		
A++	-0.180489+0.191282j	-0.180489+0.191282j	5.53e-14
A+-	0.689818-0.731068j	0.689818-0.731068j	2.67e-15
A-+	0.689818-0.731068j	0.689818-0.731068j	2.56e-15
A--	0.180489-0.191282j	0.180489-0.191282j	5.51e-14
ArD(1600)1	$\mathcal{H}^\mathrm{LS,production}_{\Delta(1600),1,\frac{3}{2}}$		
A++	-0.297624-0.084307j	-0.297624-0.084307j	1.83e-14

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Table 1.2 – continued from previous page

	Computed	Expected	Difference
A+-	0.323720+0.091699j	0.323720+0.091699j	4.47e-15
A-+	-0.323720-0.091699j	-0.323720-0.091699j	4.47e-15
A--	-0.297624-0.084307j	-0.297624-0.084307j	1.83e-14
ArD(1600)2	$\mathcal{H}^{\mathrm{LS,production}}_{\Delta(1600),2,\frac{3}{2}}$		
A++	0.143541+0.040660j	0.143541+0.040660j	5.53e-14
A+-	-0.548604-0.155402j	-0.548604-0.155402j	2.35e-15
A-+	-0.548604-0.155402j	-0.548604-0.155402j	2.20e-15
A--	-0.143541-0.040660j	-0.143541-0.040660j	5.47e-14
ArD(1700)1	$\mathcal{H}^{\mathrm{LS,production}}_{\Delta(1700),1,\frac{3}{2}}$		
A++	-0.042164-0.003922j	-0.042164-0.003922j	1.10e-13
A+-	-0.226551-0.021074j	-0.226551-0.021074j	1.47e-14
A-+	-0.226551-0.021074j	-0.226551-0.021074j	1.47e-14
A--	0.042164+0.003922j	0.042164+0.003922j	1.11e-13
ArD(1700)2	$\mathcal{H}^{\mathrm{LS,production}}_{\Delta(1700),2,\frac{3}{2}}$		
A++	-0.105349-0.009800j	-0.105349-0.009800j	5.81e-14
A+-	0.336381+0.031290j	0.336381+0.031290j	2.34e-14
A-+	-0.336381-0.031290j	-0.336381-0.031290j	2.34e-14
A--	-0.105349-0.009800j	-0.105349-0.009800j	5.81e-14
ArK(892)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),0,\frac{1}{2}}$		
A++	0.219513+2.386943j	0.219513+2.386943j	5.19e-15
A+-	-0.857733-9.326825j	-0.857733-9.326825j	3.53e-15
A-+	-0.857733-9.326825j	-0.857733-9.326825j	3.53e-15
A--	-0.219513-2.386943j	-0.219513-2.386943j	5.19e-15
ArK(892)2	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),1,\frac{1}{2}}$		
A++	0.219549+2.387337j	0.219549+2.387337j	7.53e-15
A+-	-0.857874-9.328364j	-0.857874-9.328364j	2.80e-15
A-+	0.857874+9.328364j	0.857874+9.328364j	2.80e-15
A--	0.219549+2.387337j	0.219549+2.387337j	7.53e-15
ArK(892)3	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),1,\frac{3}{2}}$		
A++	0.310489+3.376204j	0.310489+3.376204j	4.72e-15
A+-	0.606609+6.596150j	0.606609+6.596150j	2.80e-15
A-+	-0.606609-6.596150j	-0.606609-6.596150j	2.80e-15
A--	0.310489+3.376204j	0.310489+3.376204j	4.72e-15
ArK(892)4	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}$		
A++	0.310629+3.377724j	0.310629+3.377724j	1.42e-14
A+-	0.606882+6.599119j	0.606882+6.599119j	7.92e-15
A-+	0.606882+6.599119j	0.606882+6.599119j	7.92e-15
A--	-0.310629-3.377724j	-0.310629-3.377724j	1.42e-14
ArK(1430)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(1430),0,\frac{1}{2}}$		
A++	0.643091+0.051436j	0.643091+0.051436j	1.08e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.643091+0.051436j	0.643091+0.051436j	1.08e-16
ArK(1430)2	$\mathcal{H}^{\mathrm{LS,production}}_{K(1430),1,\frac{1}{2}}$		
A++	-0.643091-0.051436j	-0.643091-0.051436j	2.09e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.643091+0.051436j	0.643091+0.051436j	2.09e-16
ArK(700)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(700),0,\frac{1}{2}}$		
A++	-1.070937+2.282902j	-1.070937+2.282902j	3.94e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	

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Table 1.2 – continued from previous page

	Computed	Expected	Difference
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-1.070937+2.282902j	-1.070937+2.282902j	3.94e-16
ArK(700)2	$\mathcal{H}^{\text{LS,production}}_{K(700),1,\frac{1}{2}}$		
A++	1.070937-2.282902j	1.070937-2.282902j	4.40e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-1.070937+2.282902j	-1.070937+2.282902j	4.40e-16
ArL(1405)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1405),0,\frac{1}{2}}$		
A++	-0.398444+0.097295j	-0.398444+0.097295j	8.48e-16
A+-	-0.009584+0.002340j	-0.009584+0.002340j	7.95e-14
A-+	0.009584-0.002340j	0.009584-0.002340j	8.06e-14
A--	-0.398444+0.097295j	-0.398444+0.097295j	8.48e-16
ArL(1405)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1405),1,\frac{1}{2}}$		
A++	0.163270-0.039869j	0.163270-0.039869j	2.06e-14
A+-	0.311387-0.076037j	0.311387-0.076037j	2.48e-14
A-+	0.311387-0.076037j	0.311387-0.076037j	2.50e-14
A--	-0.163270+0.039869j	-0.163270+0.039869j	2.06e-14
ArL(1520)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1520),1,\frac{3}{2}}$		
A++	0.117387-0.135999j	0.117387-0.135999j	3.04e-14
A+-	-0.599627+0.694701j	-0.599627+0.694701j	1.89e-14
A-+	-0.599627+0.694701j	-0.599627+0.694701j	1.90e-14
A--	-0.117387+0.135999j	-0.117387+0.135999j	3.03e-14
ArL(1520)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1520),2,\frac{3}{2}}$		
A++	0.330006-0.382330j	0.330006-0.382330j	7.41e-14
A+-	0.278127-0.322225j	0.278127-0.322225j	7.88e-14
A-+	-0.278127+0.322225j	-0.278127+0.322225j	7.87e-14
A--	0.330006-0.382330j	0.330006-0.382330j	7.41e-14
ArL(1600)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1600),0,\frac{1}{2}}$		
A++	-0.431782+0.475775j	-0.431782+0.475775j	1.51e-15
A+-	0.110199-0.121426j	0.110199-0.121426j	9.38e-15
A-+	0.110199-0.121426j	0.110199-0.121426j	9.90e-15
A--	0.431782-0.475775j	0.431782-0.475775j	1.42e-15
ArL(1600)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1600),1,\frac{1}{2}}$		
A++	0.102310-0.112734j	0.102310-0.112734j	3.06e-14
A+-	-0.389148+0.428797j	-0.389148+0.428797j	2.20e-14
A-+	0.389148-0.428797j	0.389148-0.428797j	2.21e-14
A--	0.102310-0.112734j	0.102310-0.112734j	3.06e-14
ArL(1670)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1670),0,\frac{1}{2}}$		
A++	-0.817566+0.061827j	-0.817566+0.061827j	1.60e-16
A+-	-0.019666+0.001487j	-0.019666+0.001487j	7.55e-14
A-+	0.019666-0.001487j	0.019666-0.001487j	7.62e-14
A--	-0.817566+0.061827j	-0.817566+0.061827j	1.60e-16
ArL(1670)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1670),1,\frac{1}{2}}$		
A++	0.345271-0.026110j	0.345271-0.026110j	1.85e-14
A+-	0.658498-0.049798j	0.658498-0.049798j	2.38e-14
A-+	0.658498-0.049798j	0.658498-0.049798j	2.36e-14
A--	-0.345271+0.026110j	-0.345271+0.026110j	1.87e-14
ArL(1690)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1690),1,\frac{3}{2}}$		
A++	0.110308-0.071511j	0.110308-0.071511j	2.95e-14
A+-	-0.563468+0.365287j	-0.563468+0.365287j	1.82e-14
A-+	-0.563468+0.365287j	-0.563468+0.365287j	1.80e-14

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Table 1.2 – continued from previous page

	Computed	Expected	Difference
A--	-0.110308+0.071511j	-0.110308+0.071511j	2.97e-14
ArL(1690)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1690),2,\frac{3}{2}}$		
A++	0.333287-0.216064j	0.333287-0.216064j	7.61e-14
A+-	0.280891-0.182097j	0.280891-0.182097j	8.08e-14
A-+	-0.280891+0.182097j	-0.280891+0.182097j	8.08e-14
A--	0.333287-0.216064j	0.333287-0.216064j	7.61e-14
ArL(2000)1	$\mathcal{H}^{\text{LS,production}}_{\Lambda(2000),0,\frac{1}{2}}$		
A++	1.036314+1.105950j	1.036314+1.105950j	1.14e-15
A+-	0.024928+0.026603j	0.024928+0.026603j	7.76e-14
A-+	-0.024928-0.026603j	-0.024928-0.026603j	7.71e-14
A--	1.036314+1.105950j	1.036314+1.105950j	1.24e-15
ArL(2000)2	$\mathcal{H}^{\text{LS,production}}_{\Lambda(2000),1,\frac{1}{2}}$		
A++	-0.529297-0.564863j	-0.529297-0.564863j	1.87e-14
A+-	-1.009471-1.077303j	-1.009471-1.077303j	2.34e-14
A-+	-1.009471-1.077303j	-1.009471-1.077303j	2.34e-14
A--	0.529297+0.564863j	0.529297+0.564863j	1.87e-14

1.3 Intensity distribution

The complete intensity expression contains **43,198 mathematical operations**.

1.3.1 Definition of free parameters

After substituting the parameters that are not production couplings, the total intensity expression contains **9,516 operations**.

1.3.2 Distribution

```
<Figure size 1644x1400 with 2 Axes>
```

Comparison with Figure 2 from the original LHCb study [1]:

```
<IPython.core.display.HTML object>
```

Tip

High-resolution versions of these plots:

- This study (left)
- Original amplitude analysis (right)

```
<Figure size 640x480 with 2 Axes>
```

```
<Figure size 1200x350 with 3 Axes>
```

`![](_images/intensity-distributions-1D.svg)`

1.3.3 Decay rates

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
<Figure size 900x900 with 1 Axes>
```

![](_images/rate-matrix.svg

1.3.4 Dominant decays

```
<Figure size 910x700 with 1 Axes>
```

![](_images/sub-regions.svg

```
<Figure size 900x900 with 1 Axes>
```

![](_images/rate-matrix-sub-region.svg

1.4 Polarimeter vector field

```
/home/runner/work/polarimetry/polarimetry/.venv/lib/python3.12/site-packages/
↪svgutils/compose.py:379: SyntaxWarning: invalid escape sequence '\.'
  m = re.match("([0-9]+\.?[0-9]*)([a-z]+)", measure)
```

ID	Final state / recoil	Subsystem / resonance
1	p	$K^{**} \to \pi^+ K^-$
2	π^+	$\Lambda^{**} \to K^- p$
3	K^-	$\Delta^{**} \to p\,\pi^+$

1.4.1 Dominant contributions

```
<Figure size 3000x2520 with 24 Axes>
```

```
CPU times: user 36 s, sys: 525 ms, total: 36.5 s
Wall time: 33.9 s
```

```
<Figure size 2600x1000 with 4 Axes>
```

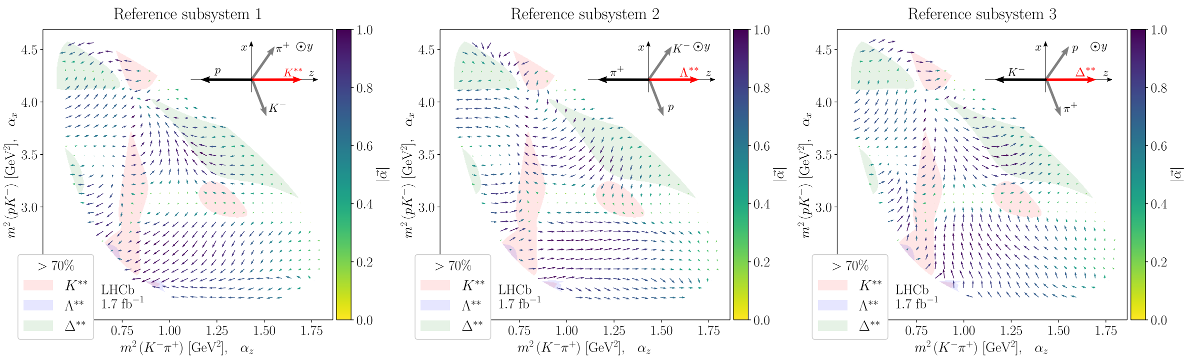
1.4.2 Total polarimetry vector field

```
  0%|          | 0/3 [00:00<?, ?it/s]
```

```
<IPython.core.display.SVG object>
```

```
<IPython.core.display.SVG object>
```

```
<IPython.core.display.SVG object>
```



1.4.3 Aligned vector fields per chain

```
<Figure size 1300x500 with 4 Axes>
```

```
<Figure size 1300x900 with 8 Axes>
```

```
<Figure size 1300x500 with 4 Axes>
```

![](_static/images/polarimetry-K-chains.svg ![](_static/images/polarimetry-L-chains.svg ![](_static/images/polarimetry-D-chains.svg

```
<Figure size 1300x450 with 4 Axes>
```

![](_static/images/polarimetry-per-subsystem.svg

1.5 Uncertainties

1.5.1 Model loading

Of the 18 models, there are 9 with a unique expression tree.

Show number of mathematical operations per model

		Model description	n ops.
0		Default amplitude model	$43,198$
1	$=0$	Alternative amplitude model with K(892) with free mass and width	$43,198$
2	$=0$	Alternative amplitude model with L(1670) with free mass and width	$43,198$
3	$=0$	Alternative amplitude model with L(1690) with free mass and width	$43,198$
4	$=0$	Alternative amplitude model with D(1232) with free mass and width	$43,198$
5	$=0$	Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width	$43,198$
6	$=0$	Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)	$43,198$
7		Alternative amplitude model with L(1800) contribution added with free mass and width	$44,222$
8		Alternative amplitude model with L(1810) contribution added with free mass and width	$46,782$
9		Alternative amplitude model with D(1620) contribution added with free mass and width	$44,222$
10		Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution	$43,470$
11	$=0$	Alternative amplitude model with K(700) with free mass and width	$43,198$
12		Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020	$46,780$
13		Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution	$43,470$
14	$=0$	Alternative amplitude model with K(1430) with free width	$43,198$
15		Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as alpha	$43,582$
16	$=0$	Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc	$43,198$
17		Alternative amplitude model obtained using LS couplings	$110,839$

1.5.2 Statistical uncertainties

Parameter bootstrapping

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
Computing decay rates:   0%|          | 0/12 [00:00<?, ?it/s]
```

Mean and standard deviations

```
(100, 100000)
```

Distributions

```
<Figure size 3340x1600 with 10 Axes>
```

```
<Figure size 3340x1600 with 10 Axes>
```

```
<Figure size 2400x1240 with 4 Axes>
```

Comparison with default values

```
<Figure size 3460x800 with 5 Axes>
```

1.5.3 Systematic uncertainties

Mean and standard deviations

```
(18, 100000)
```

Distributions

```
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```

```
<Figure size 3340x1600 with 10 Axes>
```

```
<Figure size 2400x1380 with 4 Axes>
```

1.5.4 Uncertainty on polarimetry

For each bootstrap or alternative model i, we compute the angle between each aligned polarimeter vector $\vec{\alpha}_i$ and the one from the default model, $\vec{\alpha}_0$:

$$\cos\theta_i = \frac{\vec{\alpha}_i \cdot \vec{\alpha}_0}{|\alpha_i||\alpha_0|}.$$

The solid angle can then be computed as:

$$\delta\Omega = \int_0^{2\pi} \int_0^{\theta} \mathrm{d}\phi \, \mathrm{d}\cos\theta = 2\pi \left(1 - \cos\theta\right).$$

The statistical uncertainty is given by taking the standard deviation on the $\delta\Omega$ distribution and the systematic uncertainty is given by taking finding $\theta_{\text{max}} = \max\theta_i$ and computing $\delta\Omega_{\text{max}}$ from that.

```
<Figure size 2200x800 with 4 Axes>
```

```
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```

```
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```

1.5.5 Decay rates

Resonance	Decay rate	LHCb
$\Lambda(1405)$	$7.78 \pm 0.43^{+3.01}_{-2.53}$	$7.7 \pm 0.2 \pm 3.0$
$\Lambda(1520)$	$1.91 \pm 0.10^{+0.04}_{-0.24}$	$1.86 \pm 0.09 \pm 0.23$
$\Lambda(1600)$	$5.16 \pm 0.28^{+0.50}_{-1.93}$	$5.2 \pm 0.2 \pm 1.9$
$\Lambda(1670)$	$1.15 \pm 0.04^{+0.06}_{-0.29}$	$1.18 \pm 0.06 \pm 0.32$
$\Lambda(1690)$	$1.16 \pm 0.01^{+0.06}_{-0.33}$	$1.19 \pm 0.09 \pm 0.34$
$\Lambda(2000)$	$9.55 \pm 0.67^{+0.83}_{-2.26}$	$9.58 \pm 0.27 \pm 0.93$
$\Delta(1232)$	$28.73 \pm 1.34^{+1.76}_{-0.79}$	$28.6 \pm 0.29 \pm 0.76$
$\Delta(1600)$	$4.50 \pm 0.51^{+0.93}_{-1.40}$	$4.5 \pm 0.3 \pm 1.5$
$\Delta(1700)$	$3.89 \pm 0.07^{+0.94}_{-0.48}$	$3.9 \pm 0.2 \pm 0.94$
$K(700)$	$2.99 \pm 0.20^{+0.91}_{-0.59}$	$3.02 \pm 0.16 \pm 0.92$
$K(892)$	$21.95 \pm 1.24^{+0.59}_{-0.70}$	$22.14 \pm 0.23 \pm 0.64$
$K(1430)$	$14.70 \pm 0.80^{+2.78}_{-2.67}$	$14.7 \pm 0.6 \pm 2.7$

Resonance	1	2	3	4	5	6	7	8	9	10	11	12	13
$\Lambda(1405)$	$+0.11$	-0.14	-0.01	-0.33	-0.99	$+3.01$	-2.53	-0.66	-1.58	-0.43	-0.01	-1.97	-0.11
$\Lambda(1520)$	$+0.03$	$+0.00$	$+0.01$	$+0.01$	-0.24	-0.01	-0.04	-0.08	-0.06	-0.06	$+0.04$	-0.15	-0.00
$\Lambda(1600)$	-0.02	-0.09	$+0.13$	$+0.22$	$+0.50$	-0.09	-0.30	$+0.23$	-1.93	-0.46	$+0.12$	-1.85	-0.12
$\Lambda(1670)$	-0.01	$+0.06$	$+0.03$	$+0.01$	-0.01	-0.12	-0.29	-0.03	-0.11	$+0.05$	-0.01	-0.03	$+0.01$
$\Lambda(1690)$	$+0.00$	-0.00	$+0.04$	-0.13	$+0.01$	-0.06	-0.04	-0.26	-0.33	-0.08	-0.04	$+0.06$	$+0.01$
$\Lambda(2000)$	$+0.05$	$+0.10$	-0.08	-0.09	$+0.08$	-0.85	-2.26	$+0.83$	-0.93	$+0.31$	-0.23	-0.86	$+0.35$
$\Delta(1232)$	-0.27	$+0.02$	$+0.31$	$+1.76$	-0.44	-0.14	$+0.49$	-0.63	-0.77	$+0.53$	-0.31	$+0.65$	$+0.10$
$\Delta(1600)$	$+0.33$	-0.10	-0.15	-0.28	$+0.59$	-0.38	-1.40	-0.29	$+0.93$	$+0.03$	$+0.05$	-0.58	$+0.07$
$\Delta(1700)$	-0.01	$+0.03$	-0.13	$+0.07$	$+0.39$	-0.48	-0.15	$+0.82$	$+0.94$	$+0.18$	$+0.05$	$+0.75$	$+0.03$
$K(700)$	$+0.17$	-0.02	$+0.04$	$+0.75$	$+0.62$	-0.59	-0.31	-0.06	$+0.91$	$+0.56$	$+0.25$	$+0.42$	$+0.10$
$K(892)$	-0.53	-0.02	-0.12	-0.46	-0.58	$+0.55$	$+0.59$	-0.06	$+0.18$	$+0.28$	-0.16	$+0.25$	-0.01
$K(1430)$	-0.29	$+0.07$	-0.50	-0.03	$+0.18$	-0.76	$+2.78$	$+2.40$	-2.67	$+1.29$	$+0.23$	-2.27	$+0.91$

- **0**: Default amplitude model
- **1**: Alternative amplitude model with K(892) with free mass and width
- **2**: Alternative amplitude model with L(1670) with free mass and width
- **3**: Alternative amplitude model with L(1690) with free mass and width
- **4**: Alternative amplitude model with D(1232) with free mass and width
- **5**: Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width
- **6**: Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)
- **7**: Alternative amplitude model with L(1800) contribution added with free mass and width
- **8**: Alternative amplitude model with L(1810) contribution added with free mass and width
- **9**: Alternative amplitude model with D(1620) contribution added with free mass and width
- **10**: Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution
- **11**: Alternative amplitude model with K(700) with free mass and width
- **12**: Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020
- **13**: Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution
- **14**: Alternative amplitude model with K(1430) with free width
- **15**: Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as alpha
- **16**: Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc
- **17**: Alternative amplitude model obtained using LS couplings

1.5.6 Average polarimetry values

The components of the **averaged polarimeter vector** $\overline{\alpha}$ are defined as:

$$\overline{\alpha}_j = \int I_0(\tau)\,\alpha_j(\tau)\,\mathrm{d}^n\tau \Big/ \int I_0(\tau)\;\mathrm{d}^n\tau$$

The averages of the norm of $\vec{\alpha}$ are computed as follows:

- $|\overline{\alpha}| = \sqrt{\overline{\alpha_x}^2 + \overline{\alpha_y}^2 + \overline{\alpha_z}^2}$, with the statistical uncertainties added in quadrature and the systematic uncertainties by taking the same formula on the extrema values of each $\overline{\alpha_j}$
- $\overline{|\alpha|} = \sqrt{\int I_0(\tau)\,|\vec{\alpha}(\tau)|^2\,\mathrm{d}^n\tau \Big/ \int I_0(\tau)\;\mathrm{d}^n\tau}$

Cartesian coordinates:

$$\begin{array}{rcl}
\overline{\alpha_x} &=& \left(-62.6 \pm 4.5_{-14.8}^{+8.4}\right) \times 10^{-3} \\
\overline{\alpha_y} &=& \left(+8.9 \pm 8.9_{-12.7}^{+9.1}\right) \times 10^{-3} \\
\overline{\alpha_z} &=& \left(-278.0 \pm 23.7_{-40.4}^{+12.6}\right) \times 10^{-3} \\
\overline{|\alpha|} &=& \left(669.4 \pm 9.3_{-10.4}^{+15.3}\right) \times 10^{-3}
\end{array}$$

Polar coordinates:

$$\begin{array}{rcl}
\theta(\vec{\alpha}) &=& \arccos\left(\alpha_z \big/ |\alpha|\right) \\
\phi(\vec{\alpha}) &=& \pi - \mathrm{atan2}\left(\alpha_y, -\alpha_x\right)
\end{array}$$

$$\begin{array}{rcl}
|\overline{\alpha}| &=& \left(+285.1 \pm 24.0_{-13.8}^{+37.9}\right) \times 10^{-3} \\
\theta(\overline{\alpha}) &=& +2.92 \pm 0.01_{-0.04}^{+0.05}\ \mathrm{rad} \\
&=& \left(+0.929 \pm 0.002_{-0.011}^{+0.017}\right) \times \pi \\
\phi(\overline{\alpha}) &=& +3.00 \pm 0.14_{-0.09}^{+0.21}\ \mathrm{rad} \\
&=& \left(+0.955 \pm 0.045_{-0.028}^{+0.067}\right) \times \pi
\end{array}$$

Averaged polarimeter values for each model (and the difference with the default model):

Model	$\overline{\alpha}_x$	$\overline{\alpha}_y$	$\overline{\alpha}_z$	$\overline{\lvert\alpha\rvert}$	$\Delta\overline{\alpha}_x$	$\Delta\overline{\alpha}_y$	$\Delta\overline{\alpha}_z$	$\Delta\overline{\lvert\alpha\rvert}$
0	−62.6	+8.9	−278.0	669.4				
1	−61.6	+8.5	−279.4	670.7	+1.0	−0.4	−1.4	+1.3
2	−62.9	+9.1	−278.4	669.8	−0.3	+0.2	−0.5	+0.4
3	−58.4	+7.4	−276.2	667.7	+4.2	−1.5	+1.8	−1.6
4	−69.3	+9.5	−277.2	666.9	−6.6	+0.6	+0.8	−2.5
5	−70.7	+9.6	−277.4	668.7	−8.0	+0.8	+0.6	−0.6
6	−69.7	+9.1	−281.7	673.0	−7.1	+0.2	−3.8	+3.7
7	−77.4	+18.0	−305.4	671.4	−14.8	+9.1	−27.5	+2.1
8	−55.8	+10.9	−284.6	675.5	+6.8	+2.0	−6.7	+6.1
9	−66.9	+4.4	−290.4	672.8	−4.3	−4.5	−12.4	+3.5
10	−56.4	+2.4	−265.4	659.0	+6.2	−6.5	+12.6	−10.4
11	−64.7	+9.3	−278.6	670.4	−2.1	+0.4	−0.6	+1.0
12	−75.1	+1.8	−283.4	663.5	−12.5	−7.1	−5.4	−5.8
13	−61.8	+8.1	−277.3	668.8	+0.9	−0.8	+0.7	−0.6
14	−62.2	+8.7	−277.6	669.2	+0.5	−0.2	+0.4	−0.2
15	−54.2	−3.8	−318.4	684.6	+8.4	−12.7	−40.4	+15.3
16	−62.1	+8.2	−278.1	669.5	+0.5	−0.7	−0.1	+0.2
17	−58.1	+12.1	−278.6	666.5	+4.5	+3.2	−0.6	−2.9

Model	$10^3 \cdot \lvert\overline{\alpha}\rvert$	$\theta\left(\overline{\alpha}\right)/\pi$	$\phi\left(\overline{\alpha}\right)/\pi$	$10^3 \cdot \Delta\lvert\overline{\alpha}\rvert$	$\Delta\theta\left(\overline{\alpha}\right)/\pi$	$\Delta\phi\left(\overline{\alpha}\right)/\pi$
0	$+285.1$	$+0.929$	$+0.955$			
1	$+286.2$	$+0.930$	$+0.956$	$+1.1$	$+0.001$	$+0.001$
2	$+285.6$	$+0.929$	$+0.954$	$+0.5$	-0.000	-0.001
3	$+282.4$	$+0.933$	$+0.960$	-2.7	$+0.004$	$+0.005$
4	$+285.8$	$+0.921$	$+0.956$	$+0.8$	-0.007	$+0.001$
5	$+286.4$	$+0.920$	$+0.957$	$+1.4$	-0.009	$+0.002$
6	$+290.4$	$+0.922$	$+0.959$	$+5.3$	-0.007	$+0.004$
7	$+315.6$	$+0.919$	$+0.927$	$+30.5$	-0.010	-0.028
8	$+290.3$	$+0.937$	$+0.939$	$+5.2$	$+0.008$	-0.017
9	$+298.0$	$+0.928$	$+0.979$	$+12.9$	-0.001	$+0.024$
10	$+271.3$	$+0.933$	$+0.987$	-13.8	$+0.004$	$+0.031$
11	$+286.2$	$+0.927$	$+0.955$	$+1.1$	-0.002	-0.000
12	$+293.2$	$+0.918$	$+0.992$	$+8.1$	-0.011	$+0.037$
13	$+284.2$	$+0.930$	$+0.958$	-0.9	$+0.001$	$+0.003$
14	$+284.6$	$+0.929$	$+0.956$	-0.5	$+0.000$	$+0.001$
15	$+323.0$	$+0.946$	$+1.022$	$+37.9$	$+0.017$	$+0.067$
16	$+285.1$	$+0.929$	$+0.958$	-0.0	$+0.001$	$+0.003$
17	$+284.8$	$+0.933$	$+0.935$	-0.2	$+0.004$	-0.021

Tip

These values can be downloaded in serialized JSON format under *Exported distributions* (page 26).

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```

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<Figure size 1100x500 with 2 Axes>
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```

Tip

A potential explanation for the xz-correlation may be found in Section *XZ-correlations* (page 27).

1.5.7 Exported distributions

The polarimetry fields are computed for each parameter bootstrap (statistics & systematics) and for each model on lc2pkpi-polarimetry.docs.cern.ch/uncertainties.html. All combined fields can be downloaded as single compressed TAR file under lc2pkpi-polarimetry.docs.cern.ch/_static/export/polarimetry-field.json and as a single JSON file under lc2pkpi-polarimetry.docs.cern.ch/_static/export/polarimetry-field.tar.gz.

Tip

See *Import and interpolate* (page 34) for how to use these grids in an an analysis and see *Determination of polarization* (page 42) for how to use these fields to determine the polarization from a measured distribution.

1.6 Average polarimeter per resonance

1.6.1 Computations

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

1.6.2 Result and comparison

LHCb values are taken from the original study [1]:

	this study	LHCb	1	2	3	4	5	6	7	8	9	10
$K(700)$	$+63 \pm 78^{+238}_{-235}$	$+60 \pm 660 \pm 240$	-5	-14	-55	-113	-100	$+57$	-176	-235	$+238$	$+96$
$K(892)$	$+29 \pm 15^{+31}_{-17}$		$+2$	-0	$+2$	-9	-17	$+2$	-5	$+23$	$+31$	-8
$K(1430)$	$-339 \pm 28^{+139}_{-102}$	$-340 \pm 30 \pm 140$	$+3$	$+3$	-1	-2	$+45$	$+102$	$+125$	-9	-102	$+139$
$\Lambda(1405)$	$+580 \pm 31^{+278}_{-122}$	$-580 \pm 50 \pm 280$	$+14$	-7	$+3$	$+31$	-3	-30	-122	-22	$+124$	-64
$\Lambda(1520)$	$+925 \pm 8^{+16}_{-84}$	$-925 \pm 25 \pm 84$	$+7$	$+2$	$+2$	$+16$	-34	$+2$	$+8$	$+11$	$+7$	-3
$\Lambda(1600)$	$+199 \pm 51^{+499}_{-428}$	$-200 \pm 60 \pm 500$	$+10$	-5	$+14$	-5	$+21$	$+138$	$+100$	$+499$	-428	-140
$\Lambda(1670)$	$+817 \pm 16^{+73}_{-46}$	$-817 \pm 42 \pm 73$	$+9$	-10	$+12$	$+70$	-41	-5	$+73$	$+30$	$+47$	-46
$\Lambda(1690)$	$+958 \pm 8^{+27}_{-35}$	$-958 \pm 20 \pm 27$	-3	$+6$	-12	-35	-14	$+22$	$+27$	-20	$+3$	-4
$\Lambda(2000)$	$-573 \pm 9^{+124}_{-191}$	$+570 \pm 30 \pm 190$	$+9$	-1	$+12$	$+47$	-24	-45	-191	$+58$	$+85$	$+78$
$\Delta(1232)$	$+548 \pm 8^{+36}_{-27}$	$-548 \pm 14 \pm 36$	$+9$	$+0$	-9	-14	$+17$	-1	$+10$	$+36$	$+5$	-11
$\Delta(1600)$	$-502 \pm 9^{+162}_{-112}$	$+500 \pm 50 \pm 170$	$+19$	$+10$	$+6$	$+107$	-112	$+115$	$+88$	$+49$	$+162$	$+5$
$\Delta(1700)$	$+216 \pm 18^{+42}_{-75}$	$-216 \pm 36 \pm 75$	$+40$	$+4$	-0	-19	-2	$+23$	$+16$	$+42$	$+23$	-75

1.6.3 Distribution analysis

`![](_images/alpha-z-per-resonance-statistical.svg)`

`![](_images/alpha-z-per-resonance-systematics.svg)`

XZ-correlations

It follows from the definition of $\vec{\alpha}$ for a single resonance that:

$$\begin{aligned} \alpha_x &= \left|\vec{\alpha}\right| \int I_0 \sin\left(\zeta^0\right) \mathrm{d}\tau \,/ \int I_0 \, \mathrm{d}\tau \\ \alpha_z &= \left|\vec{\alpha}\right| \int I_0 \cos\left(\zeta^0\right) \mathrm{d}\tau \,/ \int I_0 \, \mathrm{d}\tau \end{aligned}$$

This means that the correlation if 100% if I_0 does not change in the bootstrap. This may explain the xz-correlation observed for $\overline{\alpha}$ over the complete decay as reported in *Average polarimetry values* (page 25).

$$I_{\Lambda(2000)} = \frac{155.425\sigma_2^2}{\left|\sigma_2(\sigma_2-3.953)+0.79i\sqrt{0.445\sigma_2^2-\sigma_2+0.18}\right|^2}$$

$$\alpha_{x,\Lambda(2000)} = -0.572\sin\left(\zeta^0_{2(1)}\right)$$

$$\alpha_{z,\Lambda(2000)} = -0.572\cos\left(\zeta^0_{2(1)}\right)$$

```
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```

```
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```

`![](_images/alpha-xz-statistics.svg)`

`![](_images/alpha-xz-systematics.svg)`

Tip

The following plots are interactive and can best be viewed on lc2pkpi-polarimetry.docs.cern.ch.

`![](_images/alpha-xz-statistics-plotly.svg)`

`![](_images/alpha-xz-systematics-plotly.svg)`

1.7 Appendix

1.7.1 Dynamics lineshapes

$$
\begin{aligned}
F_L\left(z\right) &= \begin{cases} 1 & \text{for}\: L = 0 \\ \frac{1}{\sqrt{z^{2} + 1}} & \text{for}\: L = 1 \\ \frac{1}{\sqrt{z^{4} + 3 z^{2} + 9}} & \text{for}\: L = 2 \end{cases} \\
\lambda\left(x, y, z\right) &= x^{2} - 2 x y - 2 x z + y^{2} - 2 y z + z^{2} \\
p_{m_{i},m_{j}}\left(s\right) &= \frac{\sqrt{\lambda\left(s, m_{i}^{2}, m_{j}^{2}\right)}}{2 \sqrt{s}} \\
q_{m_{0},m_{k}}\left(s\right) &= \frac{\sqrt{\lambda\left(s, m_{0}^{2}, m_{k}^{2}\right)}}{2 m_{0}} \\
\Gamma_{l_R}\left(s\right) &= \Gamma_{0} \frac{m}{\sqrt{s}} \frac{F_{l_R}\left(R p_{m_{1},m_{2}}\left(s\right)\right)^{2}}{F_{l_R}\left(R p_{m_{1},m_{2}}\left(m^{2}\right)\right)^{2}} \left(\frac{p_{m_{1},m_{2}}\left(s\right)}{p_{m_{1},m_{2}}\left(m^{2}\right)}\right)^{2 l_R + 1}
\end{aligned}
$$

Relativistic Breit-Wigner

$$
\mathcal{R}^\mathrm{BW}_{l_R,l_{\Lambda_c}}\left(s\right) = \frac{\frac{F_{l_R}\left(R_\mathrm{res} p_{m_{1},m_{2}}\left(s\right)\right)}{F_{l_R}\left(R_\mathrm{res} p_{m_{1},m_{2}}\left(m^{2}\right)\right)} \frac{F_{l_{\Lambda_c}}\left(R_{\Lambda_c} q_{m_\mathrm{top},m_\mathrm{spectator}}\left(s\right)\right)}{F_{l_{\Lambda_c}}\left(R_{\Lambda_c} q_{m_\mathrm{top},m_\mathrm{spectator}}\left(m^{2}\right)\right)} \left(\frac{p_{m_{1},m_{2}}\left(s\right)}{p_{m_{1},m_{2}}\left(m^{2}\right)}\right)^{l_R} \left(\frac{q_{m_\mathrm{top},m_\mathrm{spectator}}\left(s\right)}{q_{m_\mathrm{top},m_\mathrm{spectator}}\left(m^{2}\right)}\right)^{l_{\Lambda_c}}}{m^{2} - i m \Gamma_{l_R}\left(s\right) - s}
$$

Bugg Breit-Wigner

$$
\begin{aligned}
\mathcal{R}^\mathrm{Bugg}\left(m_{K\pi}^{2}\right) &= \frac{1}{- \frac{i \Gamma_{0} m_{0} \left(m_{K\pi}^{2} - s_{A}\right) e^{- \gamma m_{K\pi}^{2}}}{m_{0}^{2} - s_{A}} + m_{0}^{2} - m_{K\pi}^{2}} \\
s_{A} &= m_{K}^{2} - \frac{m_{\pi}^{2}}{2} \\
p_{m_{K},m_{\pi}}\left(m_{K\pi}^{2}\right) &= \frac{\sqrt{\lambda\left(m_{K\pi}^{2}, m_{K}^{2}, m_{\pi}^{2}\right)}}{2 \sqrt{m_{K\pi}^{2}}}
\end{aligned}
$$

One of the models uses a Bugg Breit-Wigner with an exponential factor:

$$
e^{- \alpha q_{m_{0},m_{1}}\left(s\right)^{2}} \mathcal{R}^\mathrm{Bugg}\left(m_{K\pi}^{2}\right)
$$

Flatté for S-waves

$$\mathcal{R}^{\text{Flatte}}\left(s\right) = \frac{1}{m^{2} - i m \left(\frac{\Gamma_{1} m p_{m_{1},m_{2}}\left(s\right)}{\sqrt{s} p_{m_{\pi},m_{\Sigma}}\left(m^{2}\right)} + \frac{\Gamma_{2} m p_{m_{\pi},m_{\Sigma}}\left(s\right)}{\sqrt{s} p_{m_{\pi},m_{\Sigma}}\left(m^{2}\right)}\right) - s}$$

where, in this analysis, we couple the $\Lambda(1405)$ resonance to the channel $\Lambda(1405) \to \Sigma^{-}\pi^{+}$.

1.7.2 DPD angles

Equation (A1) from [2]:

$$\begin{aligned}
\theta_{12} &= \operatorname{acos}\left(\frac{2\sigma_{3}\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)-\left(m_{0}^{2}-m_{3}^{2}-\sigma_{3}\right)\left(m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{3}^{2},\sigma_{3}\right)}\sqrt{\lambda\left(\sigma_{3},m_{1}^{2},m_{2}^{2}\right)}}\right)\\
\theta_{23} &= \operatorname{acos}\left(\frac{2\sigma_{1}\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)-\left(m_{0}^{2}-m_{1}^{2}-\sigma_{1}\right)\left(m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{1}^{2},\sigma_{1}\right)}\sqrt{\lambda\left(\sigma_{1},m_{2}^{2},m_{3}^{2}\right)}}\right)\\
\theta_{31} &= \operatorname{acos}\left(\frac{2\sigma_{2}\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)-\left(m_{0}^{2}-m_{2}^{2}-\sigma_{2}\right)\left(-m_{1}^{2}+m_{3}^{2}+\sigma_{2}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{2}^{2},\sigma_{2}\right)}\sqrt{\lambda\left(\sigma_{2},m_{3}^{2},m_{1}^{2}\right)}}\right)
\end{aligned}$$

Equation (A3):

$$\begin{aligned}
\hat{\theta}_{3(1)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)+\left(m_{0}^{2}+m_{1}^{2}-\sigma_{1}\right)\left(m_{0}^{2}+m_{3}^{2}-\sigma_{3}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{1}^{2},\sigma_{1}\right)}\sqrt{\lambda\left(m_{0}^{2},\sigma_{3},m_{3}^{2}\right)}}\right)\\
\hat{\theta}_{1(2)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)+\left(m_{0}^{2}+m_{1}^{2}-\sigma_{1}\right)\left(m_{0}^{2}+m_{2}^{2}-\sigma_{2}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{2}^{2},\sigma_{2}\right)}\sqrt{\lambda\left(m_{0}^{2},\sigma_{1},m_{1}^{2}\right)}}\right)\\
\hat{\theta}_{2(3)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)+\left(m_{0}^{2}+m_{2}^{2}-\sigma_{2}\right)\left(m_{0}^{2}+m_{3}^{2}-\sigma_{3}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{3}^{2},\sigma_{3}\right)}\sqrt{\lambda\left(m_{0}^{2},\sigma_{2},m_{2}^{2}\right)}}\right)
\end{aligned}$$

Equations (A7):

$$\begin{aligned}
\zeta^{1}_{1(3)} &= \operatorname{acos}\left(\frac{2m_{1}^{2}\left(-m_{0}^{2}-m_{2}^{2}+\sigma_{2}\right)+\left(m_{0}^{2}+m_{1}^{2}-\sigma_{1}\right)\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{1}^{2},\sigma_{1}\right)}\sqrt{\lambda\left(\sigma_{3},m_{1}^{2},m_{2}^{2}\right)}}\right)\\
\zeta^{1}_{2(1)} &= \operatorname{acos}\left(\frac{2m_{1}^{2}\left(-m_{0}^{2}-m_{3}^{2}+\sigma_{3}\right)+\left(m_{0}^{2}+m_{1}^{2}-\sigma_{1}\right)\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{1}^{2},\sigma_{1}\right)}\sqrt{\lambda\left(\sigma_{2},m_{1}^{2},m_{3}^{2}\right)}}\right)\\
\zeta^{2}_{2(1)} &= \operatorname{acos}\left(\frac{2m_{2}^{2}\left(-m_{0}^{2}-m_{3}^{2}+\sigma_{3}\right)+\left(m_{0}^{2}+m_{2}^{2}-\sigma_{2}\right)\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{2}^{2},\sigma_{2}\right)}\sqrt{\lambda\left(\sigma_{1},m_{2}^{2},m_{3}^{2}\right)}}\right)\\
\zeta^{2}_{3(2)} &= \operatorname{acos}\left(\frac{2m_{2}^{2}\left(-m_{0}^{2}-m_{1}^{2}+\sigma_{1}\right)+\left(m_{0}^{2}+m_{2}^{2}-\sigma_{2}\right)\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{2}^{2},\sigma_{2}\right)}\sqrt{\lambda\left(\sigma_{3},m_{2}^{2},m_{1}^{2}\right)}}\right)\\
\zeta^{3}_{3(2)} &= \operatorname{acos}\left(\frac{2m_{3}^{2}\left(-m_{0}^{2}-m_{1}^{2}+\sigma_{1}\right)+\left(m_{0}^{2}+m_{3}^{2}-\sigma_{3}\right)\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{3}^{2},\sigma_{3}\right)}\sqrt{\lambda\left(\sigma_{2},m_{3}^{2},m_{1}^{2}\right)}}\right)\\
\zeta^{3}_{1(3)} &= \operatorname{acos}\left(\frac{2m_{3}^{2}\left(-m_{0}^{2}-m_{2}^{2}+\sigma_{2}\right)+\left(m_{0}^{2}+m_{3}^{2}-\sigma_{3}\right)\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)}{\sqrt{\lambda\left(m_{0}^{2},m_{3}^{2},\sigma_{3}\right)}\sqrt{\lambda\left(\sigma_{1},m_{3}^{2},m_{2}^{2}\right)}}\right)
\end{aligned}$$

Equations (A10):

$$\begin{aligned}
\zeta^{1}_{2(3)} &= \operatorname{acos}\left(\frac{2m_{1}^{2}\left(m_{2}^{2}+m_{3}^{2}-\sigma_{1}\right)+\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)}{\sqrt{\lambda\left(\sigma_{2},m_{3}^{2},m_{1}^{2}\right)}\sqrt{\lambda\left(\sigma_{3},m_{1}^{2},m_{2}^{2}\right)}}\right)\\
\zeta^{2}_{3(1)} &= \operatorname{acos}\left(\frac{2m_{2}^{2}\left(m_{1}^{2}+m_{3}^{2}-\sigma_{2}\right)+\left(-m_{1}^{2}-m_{2}^{2}+\sigma_{3}\right)\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)}{\sqrt{\lambda\left(\sigma_{1},m_{2}^{2},m_{3}^{2}\right)}\sqrt{\lambda\left(\sigma_{3},m_{1}^{2},m_{2}^{2}\right)}}\right)\\
\zeta^{3}_{1(2)} &= \operatorname{acos}\left(\frac{2m_{3}^{2}\left(m_{1}^{2}+m_{2}^{2}-\sigma_{3}\right)+\left(-m_{1}^{2}-m_{3}^{2}+\sigma_{2}\right)\left(-m_{2}^{2}-m_{3}^{2}+\sigma_{1}\right)}{\sqrt{\lambda\left(\sigma_{1},m_{2}^{2},m_{3}^{2}\right)}\sqrt{\lambda\left(\sigma_{2},m_{3}^{2},m_{1}^{2}\right)}}\right)
\end{aligned}$$

1.7.3 Phase space sample

Definition

See also

AmpForm's Kinematics page.

$$\begin{cases} 1 & \text{for } \phi\left(\sigma_i, \sigma_j\right) \leq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

$$\begin{aligned} \phi\left(\sigma_i, \sigma_j\right) &= \lambda\left(\lambda\left(\sigma_j, m_j^2, m_0^2\right), \lambda\left(\sigma_k, m_k^2, m_0^2\right), \lambda\left(\sigma_i, m_i^2, m_0^2\right)\right) \\ \lambda\left(x, y, z\right) &= x^2 - 2xy - 2xz + y^2 - 2yz + z^2 \\ \sigma_k &= m_0^2 + m_1^2 + m_2^2 + m_3^2 - \sigma_i - \sigma_j \end{aligned}$$

Visualization

$$\begin{aligned} m_0 &= 2.28646 \\ m_1 &= 0.938272046 \\ m_2 &= 0.13957018 \\ m_3 &= 0.49367700000000003 \end{aligned}$$

```
<Figure size 600x500 with 1 Axes>
```

```
![](../_images/phase-space-boundary.svg)
```

```
Generating intensity-based sample:   0%|          | 0/10000000 [00:00<?, ?it/s]
```

```
<Figure size 3000x980 with 3 Axes>
```

1.7.4 Alignment consistency

$$\sum_{\lambda_0=-1/2}^{1/2} \sum_{\lambda_1=-1/2}^{1/2} \left|\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{1(1)}\right) + A^2_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{2(1)}\right) + A^3_{\lambda\ldots}\right.$$

$$\sum_{\lambda_0=-1/2}^{1/2} \sum_{\lambda_1=-1/2}^{1/2} \left|\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{1(2)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{1(2)}\right) + A^2_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{2(2)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{2(2)}\right) + A^3_{\lambda\ldots}\right.$$

$$\sum_{\lambda_0=-1/2}^{1/2} \sum_{\lambda_1=-1/2}^{1/2} \left|\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{1(3)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{1(3)}\right) + A^2_{\lambda_0', \lambda_1', 0, 0} d^{\frac{1}{2}}_{\lambda_1', \lambda_1}\left(\zeta^1_{2(3)}\right) d^{\frac{1}{2}}_{\lambda_0, \lambda_0'}\left(\zeta^0_{2(3)}\right) + A^3_{\lambda\ldots}\right.$$

See *DPD angles* (page 29) for the definition of each $\zeta^i_{j(k)}$.

Note that a change in reference sub-system requires the production couplings for certain sub-systems to flip sign:

- **Sub-system 2** as reference system: flip signs of $\mathcal{H}^\text{production}_{K^{**}}$ and $\mathcal{H}^\text{production}_{L^{**}}$
- **Sub-system 3** as reference system: flip signs of $\mathcal{H}^\text{production}_{K^{**}}$ and $\mathcal{H}^\text{production}_{D^{**}}$

```
{1: Array(2.45658113e+09, dtype=float64),
 2: Array(2.45658113e+09, dtype=float64),
 3: Array(2.45658113e+09, dtype=float64)}
```

```
<Figure size 4000x1200 with 4 Axes>
```

1.7.5 Benchmarking

Tip

This notebook benchmarks JAX on a **single CPU core**. Compare with Julia results as reported in ComPWA/polarimetry#27. See also the Extended benchmark #68 discussion.

Note

This notebook uses only one run and one loop for `%timeit`, because JAX seems to cache its return values.

```
Physical cores: 2
Total cores: 4
```

```
CPU times: user 7.53 s, sys: 92.8 ms, total: 7.62 s
Wall time: 7.63 s
```

`DataTransformer` performance

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
258 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
2.87 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
435 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

```
292 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
423 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
186 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

Parametrized function

Compare *All parameters substituted* (page 33).

Total number of mathematical operations:

- α_x: 133,630
- α_y: 133,634
- α_z: 133,630
- I_{tot}: 43,198

```
CPU times: user 8.54 s, sys: 24 ms, total: 8.57 s
Wall time: 8.57 s
```

One data point

JIT-compilation

```
<TimeitResult : 809 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 3.56 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Compiled performance

```
<TimeitResult : 1.05 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.44 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

54x54 grid sample

Compiled but uncached

```
<TimeitResult : 1.27 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.8 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 587 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.38 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

100.000 event phase space sample

Compiled but uncached

```
<TimeitResult : 1.22 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.86 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 2.52 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.99 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Recompilation after parameter modification

Compiled but uncached

```
<TimeitResult : 1.31 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.84 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 2.33 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 4.12 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

All parameters substituted

Compare *Parametrized function* (page 31).

Number of mathematical operations after substituting all parameters:

- α_x: 29,552
- α_y: 29,556
- α_z: 29,552
- I_{tot}: 9,624

```
CPU times: user 3.82 s, sys: 44 ms, total: 3.87 s
Wall time: 3.88 s
```

One data point

JIT-compilation

```
<TimeitResult : 497 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 2.98 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Compiled performance

```
<TimeitResult : 143 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 383 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

54x54 grid sample

Compiled but uncached

```
<TimeitResult : 811 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.04 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 164 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 367 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

100.000 event phase space sample

Compiled but uncached

```
<TimeitResult : 774 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 1.11 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 117 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 145 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Summary

```
<pandas.io.formats.style.Styler at 0x7f9045041070>
```

1.7.6 Polarimeter field serialization

File size checks

File sizes for **100x100 grid**:

File type	Size
`export/alpha-x-arrays.json`	**141 kB**
`export/alpha-x-pandas.json`	**311 kB**
`export/alpha-x-python.json`	**260 kB**
`export/alpha-x-pandas-json.zip`	**51 kB**
`export/alpha-x-pandas.csv`	**129 kB**

Export polarimetry grids

Decided to use the `alpha-x-arrays.json` format. It can be exported with *`export_polarimetry_field()`* (page 52).

Polarimetry grid can be downloaded here: `export/polarimetry-model-0.json` (540 kB).

Import and interpolate

The arrays in the *exported JSON files* (page 26) can be used to create a `RegularGridInterpolator` for the intensity and for each components of $\vec{\alpha}$.

> *`import_polarimetry_field()`* (page 52) returns JAX arrays, which are read-only. `RegularGridInterpolator` requires modifiable arrays, so we convert them to NumPy.
>
> Also note that the `values` array needs to be **transposed**!

This is a function that can compute an interpolated value of each of these observables for a random point on the Dalitz plane.

```
array([0.18379986])
```

Note that `RegularGridInterpolator` is already in vectorized form, so there is no need to `vectorize` it.

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
array([2165.82154945, 5481.04128781, 6254.96174147, ..., 1369.40657535,
       4456.44114915, 7197.97782088], shape=(100000,))
```

```
<Figure size 3000x2300 with 15 Axes>
```

Note

The interpolated values over this phase space sample have been visualized by interpolating again over a `meshgrid` with `scipy.interpolate.griddata`.

Tip

Determination of polarization (page 42) shows how this interpolation method can be used to determine the polarization $\vec{P}$ from a given intensity distribution.

1.7.7 Model serialization

This page demonstrates a strategy for exporting an amplitude model with its suggested parameter defaults to disk and loading it back into memory later on for computations with the computational backend.

Export model

```
No cache file /home/runner/.cache/ampform/89/9c95054769fee4f5388beaaa5edeb7,␣
↪performing xreplace()...
```

Import model

The model is saved in a Python `dict` and to a `pickle` file. The dictionary contains a SymPy expressions for the model and suggested parameter default values. These parameter and variable symbols are substituted using the `fully_substitute()` function.

Compilation

The resulting symbolic expression depends on two variables:

- $\sigma_1 = m^2_{K\pi}$, mass of the $K^-\pi^+$ system, and
- $\sigma_2 = m^2_{pK}$, mass of the pK^- system.

This expression is turned into a numerical function by either `lambdify()`, using JAX as a computational backend.

For `sympy` backend the position argument are used.

```
(np.float64(1156.5307379422927), np.float64(636.1087670531597))
```

The compilation to JAX is facilitated by `tensorwaves`:

```
Array([1156.53073794,  636.10876705], dtype=float64)
```

Serialization with `srepr`

SymPy expressions can directly be serialized to Python code as well, with the function `srepr()`. For the full intensity expression, we can do so with:

```
CPU times: user 2.2 s, sys: 4 ms, total: 2.2 s
Wall time: 2.2 s
```

This serializes the intensity expression of 43,198 nodes to a string of **1.05 MB**.

```
Add(Pow(Abs(Add(Mul(Add(Mul(Integer(-1), Pow(Add(Mul(Integer(-1), I, ...
```

It is up to the user, however, to import the classes of each exported node before the string can be unparsed with `eval()` (see this comment).

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In[16], line 1
----> 1 eval(eval_str)

File <string>:1

NameError: name 'Add' is not defined
```

In the case of this intensity expression, it is sufficient to import all definition from the main `sympy` module and the `Str` class.

```
CPU times: user 2.81 s, sys: 54 ms, total: 2.86 s
Wall time: 2.87 s
```

Notice how the imported expression is **exactly the same** as the serialized one, including assumptions:

Optionally, the `import` statements can be embedded into the string. The parsing is then done with `exec()` instead:

See `exported_intensity_model.py` for the exported model.

```
CPU times: user 369 ms, sys: 11 ms, total: 380 ms
Wall time: 379 ms
```

> **Note**
>
> The load time is faster due to caching within SymPy.

Python package

pypi package 0.0.11 | python 3.8 | 3.9 | 3.10 | 3.11 | 3.12

As noted on the main page, the source code for this analysis is available as a Python package on PyPI and can be installed as follows.

```
pip install polarimetry-lc2pkpi
```

Each of the models can then simply be imported as

$$\sum_{\lambda_0=-1/2}^{1/2} \sum_{\lambda_1=-1/2}^{1/2} \left| \sum_{\lambda_0^{\prime}=-1/2}^{1/2} \sum_{\lambda_1^{\prime}=-1/2}^{1/2} A^{1}_{\lambda_0^{\prime},\lambda_1^{\prime},0,0} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda_1}\left(\zeta^{1}_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0^{\prime}}\left(\zeta^{0}_{1(1)}\right) + A^{2}_{\lambda_0^{\prime},\lambda_1^{\prime},0,0} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda_1}\left(\zeta^{1}_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0^{\prime}}\left(\zeta^{0}_{2(1)}\right) + A^{3}_{\lambda_0^{\prime},\lambda_1^{\prime},0,0} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda}\right.$$

The expressions have to be converted to a numerical function to evaluate them over larger data samples. There are several ways of doing this (such as algebraically substituting the parameter values first), but it depends on your application what is best. Here's a small example where we want to evaluate the model over a set of data points on the Dalitz plane. We first 'unfold' the main intensity expression and lambdify it to a numerical function with JAX as computational backend.

```
No cache file /home/runner/.cache/ampform/bf/ced37d7806069a69af9ed42dfd6b2d,␣
↪performing lambdify()...
```

Now, let's say we have some data sample containing generated phase space data points in the Dalitz plane.

```
    msq_piK    msq_Kp
0  1.303328  3.505217
1  0.424415  4.382733
2  1.694020  2.827971
3  1.260368  3.106184
4  1.171106  3.424632
5  1.556747  3.210350
6  1.615608  2.484730
7  0.649837  3.523832
```

Here, we have data points for the two Mandelstam variables σ_1 and σ_2. The `invariants` attribute of the amplitude model provides symbolic expressions for how to compute the third Mandelstam. In combination with the `variables` and `parameter_defaults` attributes, we can create a data transformer for computing helicity angles and DPD alignment angles.

$$
\begin{aligned}
\sigma_1 &= m_0^2 + m_1^2 + m_2^2 + m_3^2 - \sigma_2 - \sigma_3 \\
\sigma_2 &= m_0^2 + m_1^2 + m_2^2 + m_3^2 - \sigma_1 - \sigma_3 \\
\sigma_3 &= m_0^2 + m_1^2 + m_2^2 + m_3^2 - \sigma_1 - \sigma_2
\end{aligned}
$$

Finally, we can create an input `DataSample` that we can feed to the numerical function for the amplitude model.

```
Array([2158.16752493, 5447.08091414, 6232.40502307,  681.74714874,
        902.7663634 , 5626.31808739, 7225.75775956, 2247.93599152],
 ↪dtype=float64)
```

1.7.8 Amplitude model with LS-couplings

Model inspection

$$
\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^{1}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{1(1)}\right) + A^{2}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^{0}_{2(1)}\right) + A^{3}_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^{1}_{3(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'} (
$$

$$
\begin{aligned}
A^{1}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} - \frac{\sqrt{10}\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_1) C^{\frac{3}{2},\lambda_R+\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},\frac{1}{2}} C^{\frac{1}{2},\lambda_R+\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R+\frac{1}{2}} \mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} - \frac{\sqrt{2}\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}(\sigma_1 \cdots}{} \\
A^{2}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} - \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Lambda(1520),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,-\frac{1}{2}} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} - \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) \cdots}{} \\
A^{3}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Delta(1232),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),-\frac{1}{2},0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C \cdots}{} \\
A^{1}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \frac{\sqrt{10}\delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_1) C^{\frac{3}{2},\lambda_R-\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},-\frac{1}{2}} C^{\frac{1}{2},\lambda_R-\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R-\frac{1}{2}} \mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} \frac{\sqrt{2}\delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}(\sigma_1) C \cdots}{} \\
A^{2}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Lambda(1520),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,\frac{1}{2}} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C \cdots}{} \\
A^{3}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Delta(1232),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),\frac{1}{2},0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C \cdots}{} \\
A^{1}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} - \frac{\sqrt{10}\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_1) C^{\frac{3}{2},\lambda_R+\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},\frac{1}{2}} C^{\frac{1}{2},\lambda_R+\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R+\frac{1}{2}} \mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} - \frac{\sqrt{2}\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}(\sigma_1) C \cdots}{} \\
A^{2}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} - \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Lambda(1520),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,-\frac{1}{2}} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} - \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C \cdots}{} \\
A^{3}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Delta(1232),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),-\frac{1}{2},0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C \cdots}{} \\
A^{1}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \frac{\sqrt{10}\delta_{\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_1) C^{\frac{3}{2},\lambda_R-\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},-\frac{1}{2}} C^{\frac{1}{2},\lambda_R-\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R-\frac{1}{2}} \mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{K(892),0,0} d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} \frac{\sqrt{2}\delta_{\frac{1}{2},\lambda_R-\frac{1}{2}} \mathcal{R}^{\mathrm{BW}}_{1,0}(\sigma_1) C^{\frac{1}{2},\cdots}_{0,0\cdots}}{} \\
A^{2}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Lambda(1520),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Lambda(1520),0,\frac{1}{2}} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{2,2}(\sigma_2) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R \cdots}}{} \\
A^{3}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0} C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R} \mathcal{H}^{\mathrm{LS,production}}_{\Delta(1232),2,\frac{3}{2}} \mathcal{H}^{\mathrm{decay}}_{\Delta(1232),\frac{1}{2},0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R} \mathcal{R}^{\mathrm{BW}}_{1,2}(\sigma_3) C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R \cdots}}{}
\end{aligned}
$$

Decay	coupling			factor
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1405),0,\frac{1}{2}}$	$=$	$-1.22-0.0395i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1405),1,\frac{1}{2}}$	$=$	$1.81-1.63i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1520),1,\frac{3}{2}}$	$=$	$0.192+0.167i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1520),2,\frac{3}{2}}$	$=$	$-0.116-0.243i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1600),0,\frac{1}{2}}$	$=$	$0.134+0.628i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1600),1,\frac{1}{2}}$	$=$	$1.71-1.13i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1670),0,\frac{1}{2}}$	$=$	$0.0092-0.201i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1670),1,\frac{1}{2}}$	$=$	$0.115+0.168i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1690),1,\frac{3}{2}}$	$=$	$-0.379+0.331i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(1690),2,\frac{3}{2}}$	$=$	$0.286-0.248i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(\Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(2000),0,\frac{1}{2}}$	$=$	$2.81+0.0715i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(\Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\right)\pi^+$	$\mathcal{H}^{\text{LS,production}}_{\Lambda(2000),1,\frac{1}{2}}$	$=$	$0.891+0.0874i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1232),1,\frac{3}{2}}$	$=$	$-1.5+3.16i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1232),2,\frac{3}{2}}$	$=$	$0.587-0.839i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1600),1,\frac{3}{2}}$	$=$	$1.6-2.46i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1600),2,\frac{3}{2}}$	$=$	$0.432-0.689i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(\Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1700),1,\frac{3}{2}}$	$=$	$-3.16+2.29i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(\Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+\right)K^-$	$\mathcal{H}^{\text{LS,production}}_{\Delta(1700),2,\frac{3}{2}}$	$=$	$0.179-0.299i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(700) \xrightarrow[S=0]{L=0} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(700),0,\frac{1}{2}}$	$=$	$-0.000167-0.685i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(700) \xrightarrow[S=0]{L=0} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(700),1,\frac{1}{2}}$	$=$	$-0.631+0.0404i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(892),0,\frac{1}{2}}$	$=$	1.0	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(892),1,\frac{1}{2}}$	$=$	$-0.342+0.064i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(892),1,\frac{3}{2}}$	$=$	$-0.755-0.592i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \left(K(892) \xrightarrow[S=0]{L=1} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(892),2,\frac{3}{2}}$	$=$	$-0.0938-0.38i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \left(K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(1430),0,\frac{1}{2}}$	$=$	$-1.35-3.15i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \left(K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-\right)p$	$\mathcal{H}^{\text{LS,production}}_{K(1430),1,\frac{1}{2}}$	$=$	$0.598-0.956i$	$+1$

It is asserted that these amplitude expressions to not evaluate to 0 once the Clebsch-Gordan coefficients are evaluated.

See also

See *Resonances and LS-scheme* (page 3) for the allowed LS-values.

Distribution

```
<Figure size 3000x1600 with 2 Axes>
```

```
<Figure size 2000x1600 with 2 Axes>
```

Decay rates

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

Resonance	Default	LS-model	Difference
$\Lambda(1405)$	7.78	7.02	−0.75
$\Lambda(1520)$	1.91	1.95	+0.03
$\Lambda(1600)$	5.16	5.21	+0.05
$\Lambda(1670)$	1.15	1.18	+0.02
$\Lambda(1690)$	1.16	1.09	−0.08
$\Lambda(2000)$	9.55	9.84	+0.30
$\Delta(1232)$	28.73	28.97	+0.24
$\Delta(1600)$	4.50	4.24	−0.26
$\Delta(1700)$	3.89	3.99	+0.10
$K(700)$	2.99	3.25	+0.26
$K(892)$	21.95	21.25	−0.70
$K(1430)$	14.70	15.41	+0.71

Tip

Compare with the values with uncertainties as reported in *Decay rates* (page 24).

1.7.9 PC and PV currents

```
<IPython.core.display.HTML object>
```

Model formulation

Sort couplings by PC/PV

$$\begin{aligned}
\Lambda(1405), L=0 &= \texttt{True} \\
\Lambda(1405), L=1 &= \texttt{False} \\
\Lambda(1520), L=1 &= \texttt{False} \\
\Lambda(1520), L=2 &= \texttt{True} \\
\Lambda(1600), L=0 &= \texttt{False} \\
\Lambda(1600), L=1 &= \texttt{True} \\
\Lambda(1670), L=0 &= \texttt{True} \\
\Lambda(1670), L=1 &= \texttt{False} \\
\Lambda(1690), L=1 &= \texttt{False} \\
\Lambda(1690), L=2 &= \texttt{True} \\
\Lambda(2000), L=0 &= \texttt{True} \\
\Lambda(2000), L=1 &= \texttt{False} \\
\Delta(1232), L=1 &= \texttt{True} \\
\Delta(1232), L=2 &= \texttt{False} \\
\Delta(1600), L=1 &= \texttt{True} \\
\Delta(1600), L=2 &= \texttt{False} \\
\Delta(1700), L=1 &= \texttt{False} \\
\Delta(1700), L=2 &= \texttt{True} \\
K(700), L=0 &= \texttt{True} \\
K(700), L=1 &= \texttt{False} \\
K(892), L=0 &= \texttt{False} \\
K(892), L=1 &= \texttt{True} \\
K(892), L=2 &= \texttt{False} \\
K(1430), L=0 &= \texttt{True} \\
K(1430), L=1 &= \texttt{False}
\end{aligned}$$

$$\begin{bmatrix}
K(700), L=1 & K(700), 0, -\frac{1}{2} \\
K(892), L=0 & K(700), 0, +\frac{1}{2} \\
K(892), L=2 & K(892), -1, -\frac{1}{2} \\
K(1430), L=1 & K(892), 0, -\frac{1}{2} \\
\Delta(1232), L=2 & K(892), 0, +\frac{1}{2} \\
\Delta(1600), L=2 & K(892), 1, +\frac{1}{2} \\
\Delta(1700), L=1 & K(1430), 0, -\frac{1}{2} \\
\Lambda(1405), L=1 & K(1430), 0, +\frac{1}{2} \\
\Lambda(1520), L=1 & \Delta(1232), -\frac{1}{2}, 0 \\
\Lambda(1600), L=0 & \Delta(1232), +\frac{1}{2}, 0 \\
\Lambda(1670), L=1 & \Delta(1600), -\frac{1}{2}, 0 \\
\Lambda(1690), L=1 & \Delta(1600), +\frac{1}{2}, 0 \\
\Lambda(2000), L=1 & \Delta(1700), -\frac{1}{2}, 0 \\
K(700), L=0 & \Delta(1700), +\frac{1}{2}, 0 \\
K(892), L=1 & \Lambda(1405), -\frac{1}{2}, 0 \\
K(892), L=1 & \Lambda(1405), +\frac{1}{2}, 0 \\
K(1430), L=0 & \Lambda(1520), -\frac{1}{2}, 0 \\
\Delta(1232), L=1 & \Lambda(1520), +\frac{1}{2}, 0 \\
\Delta(1600), L=1 & \Lambda(1600), -\frac{1}{2}, 0 \\
\Delta(1700), L=2 & \Lambda(1600), +\frac{1}{2}, 0 \\
\Lambda(1405), L=0 & \Lambda(1670), -\frac{1}{2}, 0 \\
\Lambda(1520), L=2 & \Lambda(1670), +\frac{1}{2}, 0 \\
\Lambda(1600), L=1 & \Lambda(1690), -\frac{1}{2}, 0 \\
\Lambda(1670), L=0 & \Lambda(1690), +\frac{1}{2}, 0 \\
\Lambda(1690), L=2 & \Lambda(2000), -\frac{1}{2}, 0 \\
\Lambda(2000), L=0 & \Lambda(2000), +\frac{1}{2}, 0
\end{bmatrix}$$

Compute decay rates

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
Computing decay rates:   0%|          | 0/351 [00:00<?, ?it/s]
```

```
Computing decay rates:   0%|          | 0/351 [00:00<?, ?it/s]
```

Visualize decay rates

SVG files can be downloaded here:

- `decay-rates-default-canonical.svg`
- `decay-rates-default-helicity.svg`

1.7.10 SU(2) → SO(3) homomorphism

The Cornwell theorem from the group theory (see for example Section 3, Chapter 5 of [3]) gives the relation between the rotation of the transition amplitude and the physical vector of polarization sensitivity:

$$R_{ij}(\phi,\theta,\chi) = \frac{1}{2}\mathrm{tr}\left(D^{1/2*}(\phi,\theta,\chi)\sigma_i^P D^{1/2*\dagger}(\phi,\theta,\chi)\sigma_j^P\right)\,, \tag{1.1}$$

where tr represents the trace operation applied to the product of the two-dimensional matrices, D and σ^P, and $R_{ij}(\phi,\theta,\chi)$ is a three-dimensional rotation matrix implementing the Euler transformation to a physical vector.

$$\begin{bmatrix} -\sin(\chi)\sin(\phi)+\cos(\chi)\cos(\phi)\cos(\theta) & -\sin(\chi)\cos(\phi)\cos(\theta)-\sin(\phi)\cos(\chi) & \sin(\theta)\cos(\phi) \\ \sin(\chi)\cos(\phi)+\sin(\phi)\cos(\chi)\cos(\theta) & -\sin(\chi)\sin(\phi)\cos(\theta)+\cos(\chi)\cos(\phi) & \sin(\phi)\sin(\theta) \\ -\sin(\theta)\cos(\chi) & \sin(\chi)\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\chi)\sin(\phi)+\cos(\chi)\cos(\phi)\cos(\theta) & -\sin(\chi)\cos(\phi)\cos(\theta)-\sin(\phi)\cos(\chi) & \sin(\theta)\cos(\phi) \\ \sin(\chi)\cos(\phi)+\sin(\phi)\cos(\chi)\cos(\theta) & -\sin(\chi)\sin(\phi)\cos(\theta)+\cos(\chi)\cos(\phi) & \sin(\phi)\sin(\theta) \\ -\sin(\theta)\cos(\chi) & \sin(\chi)\sin(\theta) & \cos(\theta) \end{bmatrix}$$

1.7.11 Determination of polarization

Given the aligned polarimeter field $\vec{\alpha}$ and the corresponding intensity distribution I_0, the intensity distribution I for a polarized decay can be computed as follows:

$$I(\phi,\theta,\chi;\tau) = I_0(\tau)\left(1+\vec{P}R(\phi,\theta,\chi)\vec{\alpha}(\tau)\right) \tag{1.2}$$

with R the rotation matrix over the decay plane orientation, represented in Euler angles (ϕ,θ,χ).

In this section, we show that it's possible to determine the polarization $\vec{P}$ from a given intensity distribution I of a λ_c decay if we the $\vec{\alpha}$ fields and the corresponding I_0 values of that Λ_c decay. We get $\vec{\alpha}$ and I_0 by interpolating the grid samples provided from *Exported distributions* (page 26) using the method described in *Import and interpolate* (page 34). We perform the same procedure with the averaged aligned polarimeter vector from Section 1.5.6 in order to quantify the loss in precision when integrating over the Dalitz plane variables τ.

Polarized test distribution

For this study, a phase space sample is uniformly generated over the Dalitz plane variables τ. The phase space sample is extended with uniform distributions over the decay plane angles (ϕ,θ,χ), so that the phase space can be used to generate a hit-and-miss toy sample for a polarized intensity distribution.

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

We now generate an intensity distribution over the phase space sample given a certain value for $\vec{P}$ [1] using Eq. (1.2) and by interpolating the $\vec{\alpha}$ and I_0 fields with the grid samples for the default model.

```
<Figure size 1500x400 with 3 Axes>
```

```
<Figure size 1200x300 with 1 Axes>
```

Using the exported polarimeter grid

The generated distribution is now assumed to be a *measured distribution* I with unknown polarization $\vec{P}$. It is shown below that the actual $\vec{P}$ with which the distribution was generated can be found by performing a fit on Eq. (1.2). This is done with `iminuit`, starting with a certain 'guessed' value for $\vec{P}$ as initial parameters.

To avoid having to generate a hit-and-miss intensity test distribution, the parameters $\vec{P} = \left(P_x, P_y, P_z\right)$ are optimized with regard to a **weighted negative log likelihood estimator**:

$$\mathrm{NLL} = -\sum_i w_i \log I_{i,\vec{P}}\left(\phi, \theta, \chi; \tau\right)\,. \tag{1.3}$$

with the normalized intensities of the generated distribution taken as weights:

$$w_i = n\,I_i\,\Big/\,\sum_j^n I_j\,, \tag{1.4}$$

such that $\sum w_i = n$. To propagate uncertainties, a fit is performed using the exported grids of each alternative model.

```
┌─────────────────────────────────────────────────────────────────────────┐
│                                Migrad                                   │
├──────────────────────────────────┬──────────────────────────────────────┤
│ FCN = 1.127e+06                  │              Nfcn = 66               │
│ EDM = 2.58e-06 (Goal: 0.0001)    │            time = 1.9 sec            │
├──────────────────────────────────┼──────────────────────────────────────┤
│          Valid Minimum           │   Below EDM threshold (goal x 10)    │
├──────────────────────────────────┼──────────────────────────────────────┤
│      No parameters at limit      │           Below call limit           │
├──────────────────────────────────┼──────────────────────────────────────┤
│             Hesse ok             │         Covariance accurate          │
└──────────────────────────────────┴──────────────────────────────────────┘
┌───┬──────┬───────────┬───────────┬────────────┬────────────┬─────────┬─────────┬───────┐
│   │ Name │   Value   │ Hesse Err │ Minos Err- │ Minos Err+ │ Limit-  │ Limit+  │ ␣
↪Fixed │
├───┼──────┼───────────┼───────────┼────────────┼────────────┼─────────┼─────────┼───────┤
│ 0 │ Px   │   0.217   │   0.008   │            │            │         │         │ ␣
↪      │
│ 1 │ Py   │   0.011   │   0.008   │            │            │         │         │ ␣
↪      │
│ 2 │ Pz   │  -0.665   │   0.007   │            │            │         │         │ ␣
↪      │
└───┴──────┴───────────┴───────────┴────────────┴────────────┴─────────┴─────────┴───────┘
┌────┬──────────────────────────────┐
│    │       Px         Py       Pz │
├────┼──────────────────────────────┤
│ Px │ 6.24e-05          0        0 │
│ Py │        0   6.27e-05        0 │
│ Pz │        0          0  5.6e-05 │
└────┴──────────────────────────────┘
```

The polarization $\vec{P}$ is determined to be (in %):

$$\begin{array}{rcl} P_x & = & +21.65^{+0.30}_{-0.62} \\ P_y & = & +1.08^{+0.02}_{-0.05} \\ P_z & = & -66.50^{+1.66}_{-0.85} \end{array}$$

with the upper and lower sign being the systematic extrema uncertainties as determined by the alternative models.

This is to be compared with the model uncertainties reported by [1]:

$$\begin{array}{rcl} P_x & = & +21.65 \pm 0.36 \\ P_y & = & +1.08 \pm 0.09 \\ P_z & = & -66.5 \pm 1.1. \end{array}$$

The polarimeter values for each model are (in %):

Model	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
0	$+21.65$	$+1.08$	-66.5			
1	$+21.59$	$+1.07$	-66.4	-0.06	-0.01	$+0.13$
2	$+21.63$	$+1.07$	-66.5	-0.02	-0.00	$+0.04$
3	$+21.69$	$+1.07$	-66.6	$+0.04$	-0.01	-0.10
4	$+21.65$	$+1.10$	-66.5	$+0.00$	$+0.02$	-0.04
5	$+21.68$	$+1.08$	-66.5	$+0.03$	$+0.01$	-0.04
6	$+21.51$	$+1.06$	-66.0	-0.14	-0.02	$+0.48$
7	$+21.18$	$+1.05$	-65.3	-0.47	-0.03	$+1.18$
8	$+21.34$	$+1.03$	-65.6	-0.31	-0.05	$+0.87$
9	$+21.34$	$+1.05$	-65.6	-0.31	-0.03	$+0.90$
10	$+21.95$	$+1.10$	-67.4	$+0.30$	$+0.02$	-0.85
11	$+21.61$	$+1.08$	-66.4	-0.04	$+0.00$	$+0.12$
12	$+21.70$	$+1.03$	-66.6	$+0.05$	-0.05	-0.10
13	$+21.67$	$+1.08$	-66.6	$+0.02$	$+0.00$	-0.05
14	$+21.66$	$+1.08$	-66.5	$+0.01$	$+0.00$	-0.02
15	$+21.03$	$+1.10$	-64.8	-0.62	$+0.02$	$+1.66$
16	$+21.64$	$+1.08$	-66.5	-0.01	$+0.00$	$+0.03$
17	$+21.67$	$+1.08$	-66.6	$+0.02$	$+0.00$	-0.09

Using the averaged polarimeter vector

Equation (1.2) requires knowledge about the aligned polarimeter field $\vec{\alpha}(\tau)$ and intensity distribution $I_0(\tau)$ over all kinematic variables τ. It is, however, also possible to compute the differential decay rate from the averaged polarimeter vector $\vec{\overline{\alpha}}$ (see *Average polarimetry values* (page 25)). The equivalent formula to Eq. (1.2) is:

$$\frac{8\pi^2}{\Gamma}\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\phi\,\mathrm{d}\cos\theta\,\mathrm{d}\chi} = 1 + \sum_{i,j} P_i R_{ij}(\phi,\theta,\chi)\overline{\alpha}_j\,, \tag{1.5}$$

We use this equation along with Eq. (1.3) to determine $\vec{P}$ with `Minuit`.

```
┌─────────────────────────────────────────────────────────────────────────┐
│                                Migrad                                   │
├──────────────────────────────────┬──────────────────────────────────────┤
│ FCN = 1.151e+06                  │              Nfcn = 56               │
│ EDM = 6.08e-08 (Goal: 0.0001)    │            time = 1.4 sec            │
├──────────────────────────────────┼──────────────────────────────────────┤
│          Valid Minimum           │   Below EDM threshold (goal x 10)    │
├──────────────────────────────────┼──────────────────────────────────────┤
│      No parameters at limit      │           Below call limit           │
├──────────────────────────────────┼──────────────────────────────────────┤
```

(continues on next page)

(continued from previous page)

```
│              Hesse ok              │         Covariance accurate         │
└────────────────────────────────────┴─────────────────────────────────────┘
┌───┬──────┬───────────┬───────────┬────────────┬────────────┬─────────┬─────────┬─
│   │ Name │   Value   │ Hesse Err │ Minos Err- │ Minos Err+ │ Limit-  │ Limit+  │␣
↪Fixed │
├───┼──────┼───────────┼───────────┼────────────┼────────────┼─────────┼─────────┼─
│ 0 │ Px   │   0.203   │   0.019   │            │            │         │         │␣
↪      │
│ 1 │ Py   │  -0.003   │   0.019   │            │            │         │         │␣
↪      │
│ 2 │ Pz   │  -0.661   │   0.019   │            │            │         │         │␣
↪      │
└───┴──────┴───────────┴───────────┴────────────┴────────────┴─────────┴─────────┴─
┌────┬─────────────────────────────┐
│    │       Px        Py       Pz │
├────┼─────────────────────────────┤
│ Px │ 0.000364        -0        0 │
│ Py │       -0  0.000367       -0 │
│ Pz │        0        -0 0.000362 │
└────┴─────────────────────────────┘
```

Using the averaged polarimeter vector $\vec{\overline{\alpha}}$, the polarization $\vec{P}$ is determined to be (in %):

$$\begin{array}{rcl} P_x &=& +20.32^{+1.04}_{-2.44} \\ P_y &=& -0.26^{+0.17}_{-0.08} \\ P_z &=& -66.14^{+7.91}_{-3.32} \end{array}\,.$$

The polarimeter values for each model are (in %):

Model	**P_x**	**P_y**	**P_z**	**⍰P_x**	**⍰P_y**	**⍰P_z**
0	$+20.32$	-0.26	-66.1			
1	$+20.23$	-0.24	-65.9	-0.08	$+0.01$	$+0.26$
2	$+20.28$	-0.26	-66.0	-0.04	-0.00	$+0.12$
3	$+20.49$	-0.22	-66.8	$+0.18$	$+0.04$	-0.63
4	$+20.29$	-0.32	-65.9	-0.03	-0.06	$+0.21$
5	$+20.25$	-0.33	-65.8	-0.07	-0.07	$+0.36$
6	$+19.97$	-0.31	-64.9	-0.35	-0.05	$+1.24$
7	$+18.34$	-0.31	-59.7	-1.98	-0.05	$+6.43$
8	$+19.90$	-0.18	-65.0	-0.42	$+0.08$	$+1.17$
9	$+19.46$	-0.25	-63.2	-0.85	$+0.01$	$+2.90$
10	$+21.36$	-0.23	-69.5	$+1.04$	$+0.03$	-3.32
11	$+20.25$	-0.28	-65.9	-0.07	-0.02	$+0.26$
12	$+19.82$	-0.34	-64.2	-0.49	-0.08	$+1.97$
13	$+20.38$	-0.25	-66.3	$+0.06$	$+0.01$	-0.20
14	$+20.35$	-0.25	-66.3	$+0.04$	$+0.00$	-0.12
15	$+17.88$	-0.09	-58.2	-2.44	$+0.17$	$+7.91$
16	$+20.32$	-0.25	-66.1	$+0.00$	$+0.01$	-0.00
17	$+20.29$	-0.22	-66.2	-0.03	$+0.04$	-0.08

Propagating extrema uncertainties

In Section 1.5.6, the averaged aligned polarimeter vectors with systematic model uncertainties were found to be:

observable	central	stat + syst
$\overline{\alpha}_x$ $[10^{-3}]$	-62.6	14.8
$\overline{\alpha}_y$ $[10^{-3}]$	$+8.9$	12.7
$\overline{\alpha}_z$ $[10^{-3}]$	-278.0	40.4
$\lvert\overline{\alpha}\rvert$ $[10^{-3}]$	285.1	37.9
$\theta(\overline{\alpha})$ $[\pi]$	$+0.929$	0.017
$\phi(\overline{\alpha})$ $[\pi]$	$+0.955$	0.067

This list of uncertainties is determined by the *extreme deviations* of the alternative models, whereas the uncertainties on the polarizations determined in Section 1.7.11 are determined by the averaged polarimeters of *all* alternative models. The tables below shows that there is a loss in systematic uncertainty when we propagate uncertainties by taking computing $\vec{P}$ *only* with combinations of $\alpha_i - \sigma_i, \alpha_i + \sigma_i$ for each $i \in x, y, z$.

```
0%|          | 0/8 [00:00<?, ?it/s]
```

```
0%|          | 0/8 [00:00<?, ?it/s]
```

Polarizations from $\overline{\alpha}$ in cartesian coordinates:

$$\begin{array}{rcrcr} P_x &=& +20.32 &\pm& 3.60 \\ P_y &=& -0.26 &\pm& 0.34 \\ P_z &=& -66.14 &\pm& 11.51 \end{array}$$

Polarizations from $\overline{\alpha}$ in polar coordinates:

$$\begin{array}{rcrcr} P_x &=& +20.32 &\pm& 3.23 \\ P_y &=& -0.26 &\pm& 0.19 \\ P_z &=& -66.14 &\pm& 10.08 \end{array}$$

α_x	α_y	α_z	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
-62.6	8.9	-278.0	$+20.32$	-0.26	-66.14			
-77.4	-3.8	-318.4	$+17.7$	-0.25	-57.4	-2.58	$+0.01$	$+8.7$
-77.4	-3.8	-237.5	$+23.3$	-0.55	-74.9	$+2.97$	-0.30	-8.7
-77.4	$+21.6$	-318.4	$+17.6$	-0.28	-57.4	-2.72	-0.02	$+8.7$
-77.4	$+21.6$	-237.5	$+23.0$	-0.60	-74.7	$+2.71$	-0.34	-8.6
-47.8	-3.8	-318.4	$+17.9$	-0.04	-58.4	-2.43	$+0.21$	$+7.8$
-47.8	-3.8	-237.5	$+23.9$	-0.21	-77.7	$+3.60$	$+0.05$	-11.5
-47.8	$+21.6$	-318.4	$+17.7$	-0.07	-58.3	-2.57	$+0.19$	$+7.8$
-47.8	$+21.6$	-237.5	$+23.6$	-0.26	-77.5	$+3.31$	$+0.00$	-11.3

$\lvert\alpha\rvert$	θ $[\pi]$	ϕ $[\pi]$	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
285.1	0.929	0.955	$+20.32$	-0.26	-66.14			
247.1	$+0.91$	$+0.89$	$+23.3$	-0.45	-76.1	$+3.01$	-0.19	-10.0
247.1	$+0.91$	$+1.02$	$+23.5$	-0.44	-75.9	$+3.23$	-0.19	-9.8
247.1	$+0.95$	$+0.89$	$+23.2$	-0.12	-76.2	$+2.91$	$+0.14$	-10.1
247.1	$+0.95$	$+1.02$	$+23.4$	-0.12	-76.1	$+3.05$	$+0.14$	-10.0
323.0	$+0.91$	$+0.89$	$+17.9$	-0.35	-58.2	-2.47	-0.09	$+7.9$
323.0	$+0.91$	$+1.02$	$+18.0$	-0.34	-58.1	-2.30	-0.08	$+8.0$
323.0	$+0.95$	$+0.89$	$+17.8$	-0.09	-58.3	-2.54	$+0.17$	$+7.8$
323.0	$+0.95$	$+1.02$	$+17.9$	-0.09	-58.2	-2.44	$+0.17$	$+7.9$

Increase in uncertainties

When the polarization is determined with the averaged aligned polarimeter vector $\vec{\overline{\alpha}}$ instead of the aligned polarimeter vector field $\vec{\alpha}(\tau)$ over all Dalitz variables τ, the uncertainty is expected to increase by a factor $S_0/\overline{S}_0 \approx 3$, with:

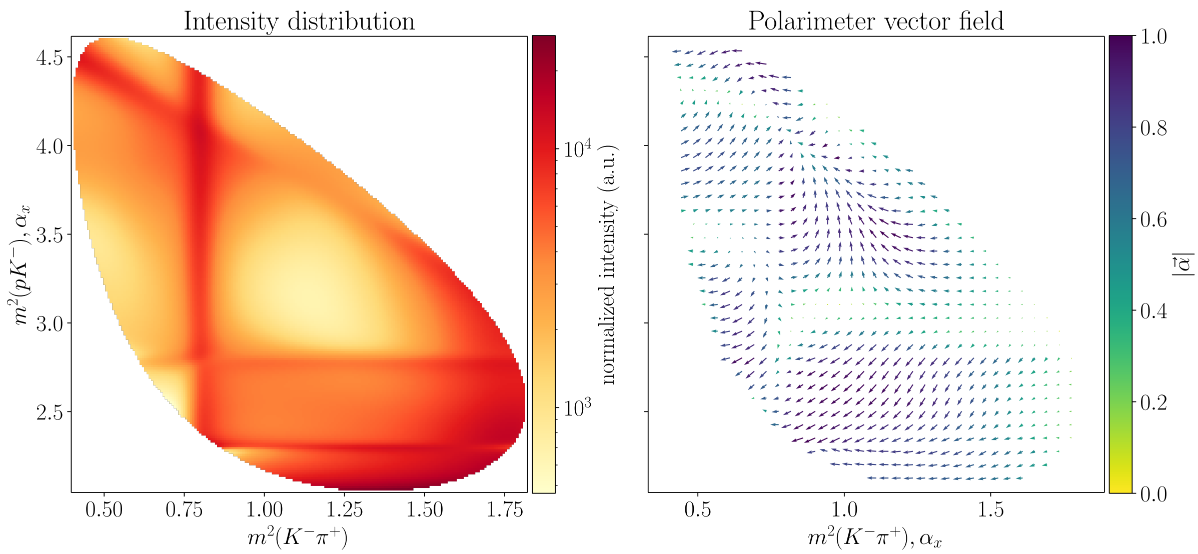
$$\begin{aligned} S_0^2 &= 3 \int I_0 \left|\vec{\alpha}\right|^2 \mathrm{d}^n\tau \Big/ \int I_0 \, \mathrm{d}^n\tau \\ \overline{S}_0^2 &= 3(\overline{\alpha}_x^2 + \overline{\alpha}_y^2 + \overline{\alpha}_z^2)\,. \end{aligned} \tag{1.6}$$

The following table shows the maximal deviation (systematic uncertainty) of the determined polarization $\vec{P}$ for each alternative model (determined with the $\overline{\alpha}$-values in cartesian coordinates). The second and third column indicate the systematic uncertainty (in %) as determined with the full vector field and with the averaged vector, respectively.

σ_{model}	$\vec{\alpha}(\tau)$	$\vec{\overline{\alpha}}$	factor
P_x	0.62	2.44	3.9
P_y	0.05	0.17	3.5
P_z	1.66	7.91	4.8

![](_static/images/polarization-distribution-systematics.svg

1.7.12 Interactive visualization



Tip

Run this notebook locally in Jupyter or online on Binder to modify parameters interactively!

1.8 Bibliography

1.9 polarimetry

```
import polarimetry
```

Main helper functions for loading the LHCb model and formulating polarimetry.

formulate_polarimetry(*builder: DalitzPlotDecompositionBuilder, reference_subsystem: FinalStateID = 1*) → tuple[PoolSum, PoolSum, PoolSum]

published_model(*model_id: int* | ModelName *(page 48) = 0, model_file: Path | str | None = None, particle_file: Path | str | None = None, cleanup_summations: bool = True*) → AmplitudeModel

Import model data and parameters, perform coupling conversions and return model.

expose_model_description() → tuple[dict[Literal['Default amplitude model', 'Alternative amplitude model with K(892) with free mass and width', 'Alternative amplitude model with L(1670) with free mass and width', 'Alternative amplitude model with L(1690) with free mass and width', 'Alternative amplitude model with D(1232) with free mass and width', 'Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width', "Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)", 'Alternative amplitude model with L(1800) contribution added with free mass and width', 'Alternative amplitude model with L(1810) contribution added with free mass and width', 'Alternative amplitude model with D(1620) contribution added with free mass and width', 'Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution', 'Alternative amplitude model with K(700) with free mass and width', 'Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020', 'Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution', 'Alternative amplitude model with K(1430) with free width', 'Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as alpha', 'Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc', 'Alternative amplitude model obtained using LS couplings'], *ModelDefinition* (page 49)], dict[str, *ResonanceJSON* (page 51)]]

Load all published model and particle definitions.

Returns a `tuple` of:

1. all 18 model definitions from `model-definitions.yaml`,
2. particle definitions from `particle-definitions.yaml`.

Submodules and Subpackages

1.9.1 lhcb

```
import polarimetry.lhcb
```

Import functions that are specifically for this LHCb analysis.

See also

Cross-check with LHCb data (page 12)

ModelName

The names of the published models.

alias of `Literal`['Default amplitude model', 'Alternative amplitude model with K(892) with free mass and width', 'Alternative amplitude model with L(1670) with free mass and width', 'Alternative amplitude model with L(1690) with free mass and width', 'Alternative amplitude model with D(1232) with free mass and width', 'Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width', "Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)", 'Alternative amplitude model with L(1800) contribution added with free mass and width', 'Alternative amplitude model with L(1810) contribution added with free mass and width', 'Alternative amplitude model with D(1620) contribution added with free mass and width', 'Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution', 'Alternative amplitude model with K(700) with free mass and width', 'Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020', 'Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution', 'Alternative amplitude model with K(1430) with free width', 'Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as alpha', 'Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc', 'Alternative amplitude model obtained using LS couplings']

LineshapeName

Allowed lineshape names in the model definition file.

alias of `Literal`['BreitWignerMinL', 'BreitWignerMinL_LS', 'BuggBreitWignerExpFF', 'BuggBreitWignerMinL', 'BuggBreitWignerMinL_LS', 'Flatte1405', 'Flatte1405_LS']

ResonanceName

Allowed resonance names in the model definition file.

alias of `Literal`['D(1232)', 'D(1600)', 'D(1620)', 'D(1700)', 'K(1410)', 'K(1430)', 'K(700)', 'K(892)', 'L(1405)', 'L(1520)', 'L(1600)', 'L(1670)', 'L(1690)', 'L(1800)', 'L(1810)', 'L(2000)']

class ModelDefinition

Bases: `TypedDict`

parameters: dict[str, str]

lineshapes: dict[Literal['D(1232)', 'D(1600)', 'D(1620)', 'D(1700)', 'K(1410)', 'K(1430)', 'K(700)', 'K(892)', 'L(1405)', 'L(1520)', 'L(1600)', 'L(1670)', 'L(1690)', 'L(1800)', 'L(1810)', 'L(2000)'], Literal['BreitWignerMinL', 'BreitWignerMinL_LS', 'BuggBreitWignerExpFF', 'BuggBreitWignerMinL', 'BuggBreitWignerMinL_LS', 'Flatte1405', 'Flatte1405_LS']]

load_model(*model_file: Path* | *str*, *particle_definitions: dict[str, Particle]*, *model_id: int* | ModelName *(page 48) = 0*, *cleanup_summations: bool = False*) → AmplitudeModel

load_model_builder(*model_file: Path* | *str*, *particle_definitions: dict[str, Particle]*, *model_id: int* | ModelName *(page 48) = 0*) → DalitzPlotDecompositionBuilder

load_three_body_decay(*resonance_names: Iterable[Literal['D(1232)', 'D(1600)', 'D(1620)', 'D(1700)', 'K(1410)', 'K(1430)', 'K(700)', 'K(892)', 'L(1405)', 'L(1520)', 'L(1600)', 'L(1670)', 'L(1690)', 'L(1800)', 'L(1810)', 'L(2000)']]*, *particle_definitions: dict[str, Particle]*, *min_ls: bool = True*) → ThreeBodyDecay

class ParameterBootstrap(*filename: Path* | *str*, *decay: ThreeBodyDecay*, *model_id: int* | ModelName *(page 48) = 0*)

Bases: `object`

A wrapper for loading parameters from `model-definitions.yaml`.

property values: dict[str, complex | int]

property uncertainties: dict[str, complex | int]

create_distribution(*sample_size: int*, *seed: int* | *None = None*) → dict[str, ndarray]

load_model_parameters(*filename: Path* | *str*, *decay: ThreeBodyDecay*, *model_id: int* | ModelName *(page 48) = 0*, *particle_definitions: dict[str, Particle]* | *None = None*) → dict[Indexed | Symbol, complex]

load_model_parameters_with_uncertainties(*filename: Path* | *str*, *decay: ThreeBodyDecay*, *model_id: int* | ModelName *(page 48) = 0*, *particle_definitions: dict[str, Particle]* | *None = None*) → dict[Indexed | Symbol, *MeasuredParameter* (page 49)]

flip_production_coupling_signs(*obj: _T*, *subsystem_names: Iterable[Literal['D', 'K', 'L']]*) → _T

compute_decay_couplings(*decay: ThreeBodyDecay*) → dict[Indexed, *MeasuredParameter* (page 49)[int]]

class ParameterType

Template for the parameter type of a for `MeasuredParameter` (page 49).

alias of TypeVar('ParameterType', complex, float, int)

class MeasuredParameter(*value: ParameterType (page 49), hesse: ParameterType (page 49), model: ParameterType (page 49) | None = None, systematic: ParameterType (page 49) | None = None*)

Bases: `Generic`[*ParameterType* (page 49)]

Data structure for imported parameter values.

MeasuredParameter.value (page 50) and *hesse* (page 50) are taken from the supplemental material, whereas *model* (page 50) and *systematic* (page 50) are taken from Tables 8 and 9 from the original LHCb paper [1].

value: *ParameterType* (page 49)

Central value of the parameter as determined by a fit with Minuit.

hesse: *ParameterType* (page 49)

Parameter uncertainty as determined by a fit with Minuit.

model: *ParameterType* (page 49) | None

Systematic uncertainties from fit bootstrapping.

systematic: *ParameterType* (page 49) | None

Systematic uncertainties from detector effects etc..

property uncertainty: *ParameterType* (page 49)

get_conversion_factor(*resonance: Particle*) → Literal[-1, 1]

get_conversion_factor_ls(*resonance: Particle, L: Rational, S: Rational*) → Literal[-1, 1]

parameter_key_to_symbol(*key: str, particle_definitions: dict[str, Particle], min_ls: bool = True*) → Indexed | Symbol

extract_particle_definitions(*decay: ThreeBodyDecay*) → dict[str, Particle]

Submodules and Subpackages

dynamics

```
import polarimetry.lhcb.dynamics
```

Dynamics lineshape definitions for the LHCb amplitude model.

formulate_bugg_breit_wigner(*decay_chain: ThreeBodyDecayChain*) → tuple[BuggBreitWigner, dict[Symbol, complex | float]]

formulate_exponential_bugg_breit_wigner(*decay_chain: ThreeBodyDecayChain*) → tuple[Expr, dict[Symbol, complex | float]]

See this paper, Eq. (4).

formulate_flatte_1405(*decay_chain: ThreeBodyDecayChain*) → tuple[FlattéSWave, dict[Symbol, complex | float]]

formulate_breit_wigner(*decay_chain: ThreeBodyDecayChain*) → tuple[BreitWignerMinL, dict[Symbol, complex | float]]

particle

```
import polarimetry.lhcb.particle
```

Hard-coded particle definitions.

load_particles(*filename: Path | str*) → dict[str, Particle]

Load Particle definitions from a YAML file.

class ResonanceJSON

Bases: TypedDict

latex: str

jp: str

mass: float | str

width: float | str

1.9.2 data

```
import polarimetry.data
```

Helper functions for importing data and creating data transformers.

create_data_transformer(*model: AmplitudeModel, backend: str = 'jax'*) → SympyDataTransformer

create_phase_space_filter(*decay: ThreeBodyDecay, x_mandelstam: FinalStateID = 1, y_mandelstam: FinalStateID = 2, outside_value=nan*) → PositionalArgumentFunction

generate_meshgrid_sample(*decay: ThreeBodyDecay, resolution: int, x_mandelstam: FinalStateID = 1, y_mandelstam: FinalStateID = 2*) → DataSample

Generate a numpy.meshgrid sample for plotting with matplotlib.pyplot.

generate_sub_meshgrid_sample(*decay: ThreeBodyDecay, resolution: int, x_range: tuple[float, float], y_range: tuple[float, float], x_mandelstam: FinalStateID = 1, y_mandelstam: FinalStateID = 2*) → DataSample

generate_phasespace_sample(*decay: ThreeBodyDecay, n_events: int, seed: int | None = None*) → DataSample

Generate a uniform distribution over Dalitz variables $\sigma_{1,2,3}$.

compute_dalitz_boundaries(*decay: ThreeBodyDecay*) → tuple[tuple[float, float], tuple[float, float], tuple[float, float]]

1.9.3 function

```
import polarimetry.function
```

Helper functions for creating numerical functions from symbolic expressions.

compute_sub_function(*func: ParametrizedFunction, input_data: DataSample, non_zero_couplings: list[str]*)

set_parameter_to_zero(*func: ParametrizedFunction, search_term: str | Pattern[str]*) → None

interference_intensity(*func, data, chain1: list[str], chain2: list[str]*) → Array

sub_intensity(*func: ParametrizedFunction, data, non_zero_couplings: list[str], axis: Axis = None*) → ndarray

integrate_intensity(*intensities, axis: Axis = None*) → ndarray

1.9.4 io

```
import polarimetry.io
```

Import-output of the polarimeter field and improvements to printing in notebooks.

display_latex(*obj*) → None

display_doit(*expr: Expr*, *deep=False*, *terms_per_line: int | None = None*) → None

mute_jax_warnings() → None

export_polarimetry_field(*sigma1: Array*, *sigma2: Array*, *alpha_x: Array*, *alpha_y: Array*, *alpha_z: Array*, *intensity: Array*, *filename: str*, *metadata: dict | None = None*) → None

import_polarimetry_field(*filename: str*, *steps: int = 1*) → dict[str, Array]

1.9.5 plot

```
import polarimetry.plot
```

Helper functions for `matplotlib`.

add_watermark(*ax: Axes*, *x: float = 0.03*, *y: float = 0.03*, *fontsize: int | None = None*, ***kwargs*) → None

get_contour_line(*contour_set: QuadContourSet*) → Artist

reduce_svg_size(*path: str*) → None

convert_svg_to_png(*input_file: str*, *dpi: int*) → None

use_mpl_latex_fonts(*reset_mpl: bool = True*) → None

Notebook execution times

Document	Modified	Method	Run Time (s)	Status
amplitude-model (page 3)	2025-03-14 19:09	cache	10.3	✅
appendix/alignment (page 30)	2025-03-14 19:10	cache	59.03	✅
appendix/angles (page 29)	2025-03-14 19:10	cache	2.44	✅
appendix/benchmark (page 31)	2025-03-14 19:11	cache	54.9	✅
appendix/dynamics (page 28)	2025-03-14 19:11	cache	2.4	✅
appendix/homomorphism (page 42)	2025-03-14 19:11	cache	2.37	✅
appendix/ls-model (page 37)	2025-03-14 19:12	cache	89.54	✅
appendix/model-serialization (page 35)	2025-03-14 19:13	cache	36.37	✅
appendix/pc-pv-currents (page 40)	2025-03-14 20:50	cache	497.68	✅
appendix/phase-space (page 30)	2025-03-14 19:22	cache	18.29	✅
appendix/serialization (page 34)	2025-03-14 19:23	cache	38.3	✅
appendix/widget (page 47)	2025-03-14 19:26	cache	220.5	✅
cross-check (page 12)	2025-03-14 19:27	cache	14.96	✅
index (page 1)	2025-03-14 19:27	cache	1.92	✅
intensity (page 19)	2025-03-14 19:31	cache	240.22	✅
polarimetry (page 20)	2025-03-14 19:33	cache	138.88	✅
resonance-polarimetry (page 27)	2025-03-14 19:48	cache	926.83	✅
uncertainties (page 21)	2025-03-14 19:57	cache	538.45	✅
zz.polarization-fit (page 42)	2025-03-14 19:59	cache	93.69	✅

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PYTHON MODULE INDEX

p

INDEX